THE EFFECTS OF MONETARY POLICY “NEWS” AND “SURPRISES”

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APPENDIX A - MODEL DERIVATION.

This section outlines the derivation of the model equations in (2.1)-(2.3) in the main text, which is now standard in the New Keynesian literature (e.g., Woodford, 2003). A similar model, without news shocks, has been estimated, for example, in Dennis (2004, 2009).

Each household solves the following optimization problem

$$\max_{C,L,B} E_0 \sum_{t=0}^{\infty} \beta^t \left[ e^{\tilde{g}_t} (C_t - \phi C_{t-1})^{1-\sigma^{-1}} - \frac{L_t^{1+\chi}}{1+\chi} \right]$$

subject to the period budget constraint

$$C_t + \frac{B_t}{P_t} = W_t L_t + \frac{(1 + R_{t-1})B_{t-1}}{P_t} + \frac{D_t}{P_t} - T_t.$$  

Each household, therefore, derives utility from consumption $C_t$ and disutility from hours of labor supplied $L_t$. The utility function is characterized by external habit formation, i.e., consumers value current consumption in relation to past aggregate consumption. The coefficient $\beta$ denotes the discount factor, $\sigma$ and $\chi$ denote the elasticities of intertemporal substitution and of labor supply, while $\phi$ measures the degree of habit formation. The term $e^{\tilde{g}_t}$ represents an aggregate shock that shifts consumers’ preferences. Expected discounted lifetime utility is maximized subject to the budget constraint (1.2), where $B_t$ denotes nominal bond holdings, $P_t$ denotes the aggregate price level, $W_t$ the nominal wage, $R_t$ the nominal interest rate, $D_t$ dividend distributions from household-owned firms, and $T_t$ are net transfers. The first order conditions imply

$$e^{\tilde{g}_t} (C_t - \phi C_{t-1})^{-\frac{1}{\sigma}} = \lambda_t$$  

$$\lambda_t = \beta (1 + R_t) (P_t/P_{t+1}) E_t \lambda_{t+1}$$

$$L_t^{\chi} = \lambda_t W_t.$$  

From (1.3) and (1.4), we obtain the Euler equation, which is then loglinearized around a zero-inflation steady state to yield

$$c_t = \frac{1}{1+\phi} E_t c_{t+1} + \frac{\phi}{1+\phi} c_{t-1} - \frac{\sigma (1-\phi)}{1+\phi} (\tilde{r}_t - E_t \pi_{t+1} - \tilde{\rho} - \Delta \tilde{g}_{t+1}).$$

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where $\tilde{\rho} = -\log \beta$ is the discount rate. Small letter variables denote log deviations from the steady state $x_t = \log(X_t/X)$; $i_t$ denotes the short-term nominal interest rate and $\pi_t$ denotes the inflation rate.

The labor supply equilibrium condition (1.5), in loglinear terms, implies
\begin{equation}
\chi l_t - w_t - \tilde{g}_t = \frac{1}{\sigma(1 - \phi)} (c_t - \phi c_{t-1}).
\end{equation}

The loglinearized Euler equation can be re-expressed in terms of the output gap, by using the resource constraint $c_t = y_t$ and the output gap definition $x_t = y_t - y_t^*$:
\begin{equation}
x_t = \frac{1}{1 + \phi} E_t x_{t+1} + \frac{\phi}{1 + \phi} x_{t-1} - \frac{\sigma(1 - \phi)}{1 + \phi} (i_t - \bar{E}_t \pi_{t+1}) + g_t
\end{equation}

with $g_t = \frac{1}{1 + \phi} [\sigma(1 - \phi) (\hat{\rho} + \Delta \tilde{g}_{t+1}) + \left( (y_{t+1} - \phi y_t^*) - (y_t^* - \phi y_{t-1}^*) \right)]$.

The production side of the economy is characterized by a continuum of monopolistically competitive firms. Prices are sticky à la Calvo: each firm has a $(1 - \alpha)$ probability of re-optimizing its price in every period. Firms that are not allowed to optimize use the indexation rule proposed by Christiano et al. (2005):
\begin{equation}
\log p_t^i = \log p_{t-1}^i + \gamma \pi_{t-1},
\end{equation}

where $\gamma$ measures the degree of indexation to past inflation.

Each firm maximizes the expected discounted stream of future profits given the demand curve for their product $y_t^i = ((p_t^i / P_{t+\tau}) (P_{t+\tau-1} / P_{t-1})^{\gamma})^{-\theta} Y_{t+\tau}$, and its production function $y_t^i = A_t (L_t)^{\gamma}$:
\begin{equation}
\max_{p_t^i} \mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{(\alpha \beta)^{\tau} \lambda_{t+\tau}}{\lambda_t} \left[ p_t^i \left( \frac{P_{t+\tau-1}}{P_{t-1}} \right)^{\gamma} \left( \frac{p_t^i}{P_{t+\tau}} \left( \frac{P_{t+\tau-1}}{P_{t-1}} \right)^{\gamma} \right)^{-\theta} Y_{t+\tau} \right. \\
- W_{t+\tau} \left( \left( \frac{p_t^i}{P_{t+\tau}} \left( \frac{P_{t+\tau-1}}{P_{t-1}} \right)^{\gamma} \right)^{-\theta} \frac{Y_{t+\tau}}{A_{t+\tau}} \right)^{1 - \gamma} \left. \right]\right],
\end{equation}

where $p_t^i$ denotes the optimal price to be chosen, $P_t$ denotes the aggregate price level, $A_t$ denotes aggregate technology, $\theta$ indicates the elasticity of substitution among differentiated products, and $\eta$ accounts for diminishing returns to scale. The first order condition can be expressed as
\begin{equation}
\mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{(\alpha \beta)^{\tau} \lambda_{t+\tau}}{\lambda_t} \left\{ (1 - \theta) (p_t^i)^{\theta-1} \left( \frac{P_{t+\tau-1}}{P_{t-1}} \right)^{\gamma} \left( \frac{1}{P_{t+\tau}} \left( \frac{P_{t+\tau-1}}{P_{t-1}} \right)^{\gamma} \right)^{-\theta} P_{t+\tau} Y_{t+\tau} \right. \\
\times \left. \left[ \frac{p_t^i}{P_{t+\tau}} \left( \frac{P_{t+\tau-1}}{P_{t-1}} \right)^{\gamma} - \left( \frac{\theta}{\theta - 1} \right) \frac{W_{t+\tau}}{P_{t+\tau} A_{t+\tau} \eta} \left( \left( \frac{P_{t+\tau-1}}{P_{t-1}} \right)^{\gamma} \right)^{-\theta} \frac{Y_{t+\tau}}{A_{t+\tau}} \right)^{1 - \gamma} \right\} \right\} = 0.
\end{equation}

The aggregate price index evolves as
\begin{equation}
P_t = \left[ (1 - \alpha) (p_t^i)^{(1-\theta)} + \alpha \left( P_{t-1} \left( \frac{P_{t-1}}{P_{t-2}} \right)^{(1-\theta)} \right) \right]^{1/(1-\theta)}.
\end{equation}
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Log-linearization of the first order conditions (1.11) and (1.12) yields

\[ p_t^* = \left( 1 - \alpha \beta \right) E_t \sum_{\tau=0}^{\infty} (\alpha \beta)^\tau \left[ \sum_{k=1}^{\tau} (\pi_{t+k} - \gamma \pi_{t+k-1}) + mc_{t+k} \right] \]  
(1.13)

\[ p_t^* = \frac{\alpha}{1 - \alpha} (\pi_t - \gamma \pi_{t-1}), \]  
(1.14)

where \( mc_t \) denotes real marginal costs and \( p_t^* \equiv \log(p_t^*/P_t) \). By quasi-differentiating (1.13) and plugging (1.14) into (1.13), we obtain the New Keynesian Phillips curve, written in terms of the economy’s aggregate marginal cost:

\[ \pi_t = \beta + \frac{\gamma}{1 + \beta \gamma} \pi_{t+1} + \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha(1 + \beta \gamma)} mc_t. \]  
(1.15)

The marginal cost is equal to the real wage minus the marginal product of labor \( mc_t = w_t - a_t - (\eta - 1) \bar{y}_t \). The real wage is equal to marginal rate of substitution between consumption and leisure, given by \( w_t = \chi l_t - \bar{y}_t + \frac{1}{\sigma(1 - \phi)} (c_t - \phi c_{t-1}) \). Plugging in the production function, we have \( w_t = \chi \eta^{-1} (c_t - a_t) - \bar{y}_t + \frac{1}{\sigma(1 - \phi)} (c_t - \phi c_{t-1}) \). Therefore, the marginal cost is given by

\[ mc_t = \left[ \omega c_t + \frac{\sigma^{-1}}{1 - \phi} (c_t - \phi c_{t-1}) - \frac{\chi + 1}{\eta} a_t - \bar{y}_t \right], \]  
(1.16)

where \( \omega = \left( \chi - (\eta - 1) \right)/\eta \). Finally, by using \( c_t = y_t \), \( x_t = y_t - y_t^* \), and the steady-state relation \( mc = 1/\mu \), equation (1.15) can be re-expressed in terms of the output gap:

\[ \pi_t = \frac{\beta}{1 + \beta \gamma} E_t \pi_{t+1} + \frac{\gamma}{1 + \beta \gamma} \pi_{t-1} + \frac{\xi}{1 + \beta \gamma} \left( \omega x_t + \frac{\sigma^{-1}}{1 - \phi} (x_t - \phi x_{t-1}) \right) + \mu_t \]  
(1.17)

where \( \xi \equiv \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \); the term \( \mu_t \) denotes a cost-push supply shock, which is sometimes simply appended to the model, but which is straightforward to derive endogenously by assuming a time-varying elasticity of substitution \( \theta_t \), instead.

REFERENCES


