Dynamic Prudential Regulation: 
Is Prompt Corrective Action Optimal?

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Abstract

The current US bank capital regulation features Prompt Corrective Action, which mandates regulators to intervene in and liquidate banks based on their book-value capital ratios. To see if Prompt Corrective Action is optimal, we build a dynamic model of repeated interactions between a banker and a regulator. Under hidden choice of risk, private information on returns and limited commitment by the banker and costly liquidation, we first characterize the optimal incentive-feasible allocation. We then demonstrate that the optimal allocation is implementable through the combination of a risk-based deposit insurance premium and a book-value capital regulation with stochastic liquidation.

Keywords: Prompt Corrective Action, Risk-based Deposit Insurance Premium, Dynamic Contracts, Mechanism Design.

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1 Introduction

After the Savings and Loan Crisis in the 1980s, US bank regulators were heavily criticized for their forbearance, i.e., not closing down failing banks sufficiently quickly. In response, the US Congress enacted the Federal Deposit Insurance Corporation Improvement Act (FDICIA) in 1991. According to Kaufman (1995), the FDICIA is viewed by many as the most important banking legislation since the Banking (Glass-Steagall) Act of 1933. The FDICIA explicitly set the goal of the regulator as “to resolve the problems of insured depository institutions at the least possible long-term loss to the deposit insurance fund”. To achieve this goal, the FDICIA introduced Prompt Corrective Action (PCA) and risk-based deposit insurance premiums. PCA links interventions in and liquidation of a bank with the level of book-value capital ratios, while the risk-based deposit insurance premium varies with capital ratios and supervisory ratings of individual banks.

In this paper, we ask if the current PCA is optimal. To answer if it is optimal to close a bank promptly or delay closure requires a dynamic model. Also, in designing banking regulation, it is natural to consider the long-term relationship between a regulator and a bank. Thus we develop a dynamic model of prudential regulation based on the following assumptions: first, in every period a banker determines his unobservable level of risk, receives private information on returns, and chooses either to close his bank and enjoy an outside option or to continue operating the bank; second, a regulator can liquidate a bank, although liquidation is socially costly. In this setup, we show that it is optimal to base bank capital regulation on book-value capital, and that it is optimal for the regulator to use stochastic liquidation/bailout rather than deterministic liquidation with no bailout as in PCA.

The model used in the analysis is a variant of DeMarzo-Fishman’s (2002) dynamic model of entrepreneurial finance. We first construct a simple dynamic economy with a risk-neutral bank owner/manager (henceforth banker) and a risk-neutral regulator (henceforth the FDIC). The FDIC is modeled as both the provider of deposit insurance and bank regulator. Its ultimate goals include promoting productive efficiency and maintaining finan-
cial system stability. In this paper, we focus on the FDIC’s objective of minimizing the potential tax burden related to the closure of banks: it is the FDIC’s obligation to make up for capital shortfalls to depositors when a bank is closed. The banker is assumed to have access to a long-term risky investment opportunity with a return whose distribution is known a priori to the FDIC. The FDIC proposes long-term regulation to the banker, including an initial level of required capital. Once the banker accepts the regulation, the FDIC charts the bank, and the banker takes deposits and invests them into the project, which generates returns that are independent over time. A return realized at time $t$ is either consumed by the banker or paid to the FDIC as a deposit insurance premium.

There are three contracting frictions in each period: first, the banker can exert effort to improve the ex-ante return distribution, but the effort is costly and unobservable; second, the banker can observe the realized return, while the FDIC only observes the return reported by the banker; third, the banker can close down the bank and enjoy an outside option, while the FDIC can also liquidate the bank and consume the proceeds from liquidation. The proceeds from liquidation are assumed to be lower than the project’s value in the absence of the contracting frictions. The incentive-feasible allocation induces the level of effort desirable to the FDIC, truth-telling of the realized return and no closure of the bank by the banker in every period. In order to induce the desirable level of effort, a feasible allocation should sufficiently reward (or punish) a good (or bad) outcome. In order to induce the banker to truthfully report the realized return, the banker’s continuation utility should change one-to-one with the reported return. Finally, to avoid the closure of the bank by the banker, the FDIC should liquidate the bank with probability one when the banker is willing to close down the bank.

Given this setup, the optimal allocation for the banker and the FDIC is characterized by (i) positive consumption by the banker and no liquidation by the FDIC when the banker’s continuation utility is above a threshold (the dividend threshold), (ii) zero consumption and positive probability of liquidation when it is below a lower threshold (the termination thres-
old), and (iii) zero consumption and no liquidation when it is between the two thresholds. When high effort is first-best optimal, the optimal allocation under the contracting frictions prescribes high effort independent of the value of the banker’s continuation utility. This is a standard result. A more interesting result is that, when low effort is first-best optimal, under certain circumstances the optimal allocation prescribes low effort when the banker’s continuation utility is high, but high effort when it is low. The latter is possible because the benefit from reducing variance by inducing high effort can be larger than that from saving the cost of high effort.

To see if the optimal allocation can be implemented by a book-value capital regulation as in the current PCA, we first define book-value capital at time \( t \) as the amount of savings accumulated by the banker up to time \( t \). Thus, book-value capital is a backward-looking measure. We then demonstrate that the optimal allocation can be implemented by the combination of a risk-based deposit insurance premium and a book-value capital regulation with stochastic liquidation/bailout of an undercapitalized bank, instead of deterministic liquidation with no bailout as in the current PCA. In this implementation, the level of book-value capital is adjusted in every period by the imposition of a deposit insurance premium such that its level replicates the movement of the banker’s continuation utility over time. The model-implied deposit insurance premium is a flow of steady payments from the bank to the FDIC, similar to a tax. The optimal deposit insurance premium increases as book-value capital decreases or as the current-period return decreases.

The intuition behind the optimality of stochastic liquidation is simple. Under deterministic liquidation, a bank is liquidated for sure when its level of book-value capital falls below a threshold. At that point, the banker’s continuation utility is zero. It is therefore difficult for the FDIC to provide the banker with any incentive to act in the FDIC’s interest. However, under stochastic liquidation, when its capital level falls below a threshold, a bank is liquidated with some probability \( p \) or bailed out with probability \( 1 - p \). The banker has positive continuation utility while he faces liquidation, and the FDIC has more room to influence the
banker’s incentives.

We also show that stochastic liquidation and partial liquidation are equivalent when we assume that partial liquidation of a bank scales down all the future cash flows. Thus, if a regulator cannot credibly implement stochastic liquidation, partial liquidation can be an alternative. Finally, comparative statics show that, when the liquidation value of a bank decreases or the riskiness of a bank increases in the sense of mean-preserving spreads, the initial required capital increases, but the deposit insurance premium schedule does not change.

This paper builds on literatures on both banking regulation and dynamic contract theory. There are few dynamic models analyzing the welfare properties of prompt corrective action. Sleet and Smith (2000) consider the appropriate design of a safety net in a two-period model when a government runs deposit insurance and a discount window. They show that, for some economies, the case for closing troubled banks “promptly” is not strong in the presence of social costs of closure. Kocherlakota and Shim (2007) construct a dynamic model economy in which entrepreneurs pledge collateral to borrow from banks. Assuming that collateral value reflects aggregate risk over time, that entrepreneurs can abscond with the project at the expense of the collateral and that depositors can withdraw deposits at any time, they show that optimal banking regulation exhibits forbearance when the ex-ante probability of a collapse in collateral values is sufficiently low, but exhibits prompt corrective action when it is sufficiently high.

The results of this paper are similar to those of Dewatripont and Tirole (1994), who analyze the effect of banks’ governance structures on managerial moral hazard in a static setting. They show that the optimal managerial incentive scheme is to threaten the manager with frequent external interference for poor performance, and reward him with a passive attitude for good performance. They also show that this policy can be implemented by both equity and debt with voluntary recapitalization. Using a mechanism design model of financial intermediation, Farhi et al (2007) show that a liquidity adequacy requirement
implements the socially optimal allocation under the unobservability of agents’ types and the possibility of hidden trades. In their setting, hidden trading on markets generates an externality which requires governmental correction.

The paper proceeds as follows. Section 2 briefly describes the current US prudential regulation. Section 3 presents the model and solves for the optimal allocation. Section 4 shows how the optimal allocation can be implemented by regulatory instruments, and also considers the possibility of using partial liquidation instead of stochastic liquidation. Section 5 provides the results of comparative statics. Section 6 compares the current US regulation with the model-implied regulation, and discusses the role of regulatory forbearance. Finally, Section 7 concludes.

2 Overview of the Current US Prudential Regulation

Prudential supervision of banks involves both government regulation and monitoring of the banking system (Mishkin 2001). Government monitoring takes the form of bank chartering before a bank opens and bank examinations thereafter. The chartering authorities screen the proposal for a new bank, examining the quality of risk management, earnings structure and the amount of initial capital, and then decide whether to charter the bank and allow it to be a member of the FDIC. In the United States, examiners give banks a CAMELS rating, where the acronym stands for the six areas assessed: capital adequacy, asset quality, management, earnings, liquidity and sensitivity to market risk. Each bank is rated from 1, the highest, to 5, the lowest, in each of the component categories, and then given a composite rating.

This paper abstracts away from examinations by assuming that the regulator depends on a periodic report by the banker on realized returns. But the paper incorporates a highly stylized type of bank chartering, assuming that the regulator knows the characteristics of the projects of a potential banker, and offers a long-term prudential regulation package, which
consists of a risk-based deposit insurance premium and a book-value capital regulation, before he charters the bank.

While monitoring takes place on an individual-bank basis, prudential regulation usually takes the form of legal rules aimed at reducing risk-taking by banks. It includes restrictions on (1) asset holdings and activities, (2) separation of banking and other financial service industries, (3) restrictions on competition, (4) capital requirements, and (5) risk-based deposit insurance premiums. The first three forms of regulation are waning nowadays, but (4) and (5) are deemed important by both practitioners and economists, hence the main focus of this paper.

The Basel scheme of bank capital regulation mandates banks to continually meet minimum capital adequacy ratios, with transfer of control following poor performance by banks and the absence of recapitalization by shareholders. The current US banking regulation stems from the FDICIA, which is based on the Basel scheme, but also has the following additional features.

The FDICIA requires risk-based deposit insurance premiums, which were introduced in 1993. The FDIC measures the risk of each bank in two dimensions: capital levels and supervisory ratings based on a bank’s composite CAMELS rating. Each dimension has three groups, so there are nine possible risk categories. The current rule, effective from 2007, consolidates the nine categories into four, and names them Risk Categories I to IV. The higher a bank’s capital adequacy and the better its CAMELS rating, the lower its insurance premium. Table 1 shows the risk categories and the deposit insurance premium rates as of the end of June 2010. This paper does not model examinations or a CAMELS rating. Instead, this paper shows that the optimal risk-based deposit insurance premium depends on the capital ratio and the current profit.

In terms of capital regulation, the FDICIA introduced early and gradual intervention rules, called PCA. Based on four different book-value capital ratios, PCA classifies banks
into five categories. Table 2 shows how US regulators classify a bank into one of the five categories.

If a bank is classified as well or adequately capitalized, it is not subject to any form of intervention. On the other hand, undercapitalized banks are subject to increasingly severe mandatory sanctions as their capital deteriorates. For all undercapitalized banks, constraints are imposed such as restrictions on distributing dividends and expanding total assets. Significantly undercapitalized banks must restore capital by selling stocks or be merged. Critically undercapitalized banks are subject to liquidation or complete asset sale. Note that PCA is a rating scheme run by the regulatory authority, so that this rating affects banks’ reputation.

Finally, in order to avoid regulatory discretion, the FDICIA introduced rigid intervention rules and public reporting of regulatory actions. In particular, it restricts discretion of the FDIC by stipulating measures the FDIC must take, according to the capital ratios of banks. Only in exceptional cases can the FDIC waive the regulatory actions required by PCA. It mandates that the supervisory agencies produce a report if a bank failure imposes costs on the FDIC, and that the report be made public and reviewed by the General Accounting Office.

3 Model

3.1 Environment

There are two infinitely-lived agents: the banker and the FDIC. Time is discrete, and time periods are indexed by \( t = 0, 1, 2, \ldots, T \). We assume infinite horizon, i.e. \( T = \infty \). There is a single perishable consumption good in every period. The banker is risk neutral, has limited wealth, and values a consumption stream \( \{c_t\}_{t=0}^{\infty} \) as \( E \left[ \sum_{t=0}^{\infty} \beta^t c_t \right] \). The FDIC is also risk neutral, has large but finite amount of wealth, and values a consumption stream \( \{x_t\}_{t=0}^{\infty} \) as \( E \left[ \sum_{t=0}^{\infty} \beta^t x_t \right] \), where \( \beta \leq \delta \).
At the beginning of period 0, the banker has an initial endowment of \( \varepsilon_0 \) units of the consumption good. If he transfers \( K_0 \) units of the good to the FDIC, he can set up a bank, receive \( D \) units of deposits from the FDIC and invest them in a long-term risky technology. We normalize \( D \) to 1, and assume \( \varepsilon_0 < 1 \) so that the banker needs to take deposits to invest. Once the size of the risky investment is fixed, it does not change during the life of the bank.

In each period \( t \geq 1 \), the banker receives a stochastic endowment from this investment, which is unobservable to the FDIC. That is, \( Y_t \) units of the good are available in period \( t \), where \( Y_t \) is continuous with an interval \( S \equiv [y, \bar{y}] \) as support, \( y \leq 0 \) and \( \bar{y} > 0 \). We assume that endowments \( \{Y_t\} \) are independent over time. The realization of the endowment good in period \( t \) is denoted by \( y_t \). The probability density of \( Y_t \), \( f^e \), is determined by the level of effort \( e \) in period \( t \). We assume that the choice of \( e_t \) only affects the current \( Y_t \), so that we preserve intertemporal independence of \( Y_t \). For simplicity, we assume that the banker can either exert high effort \( (e = 1) \) or low effort \( (e = 0) \). Exerting effort implies a disutility for the banker that is equal to \( \psi(e) \geq 0 \) units of the good, with the normalization \( \psi(e) = \psi e \), i.e., \( \psi(0) = 0 \) and \( \psi(1) = \psi > 0 \). This is equivalent to saying that the banker can exert costly monitoring effort on the firm or the project he made loans to. We assume that the banker’s utility function is separable between consumption and effort.

Costly effort is assumed to reduce risk in the sense of first-order stochastic dominance (FOSD). In particular, if \( e_t = 0 \), the distribution function of \( Y_t \) becomes \( F(y_t \mid e_t = 0) \equiv F^0(y_t) \), and if \( e_t = 1 \), it becomes \( F(y_t \mid e_t = 1) \equiv F^1(y_t) \). Note that, from the definition of FOSD, \( F^0(y) > F^1(y) \) holds for all \( y \in [y, \bar{y}] \). That is, the higher the level of effort, the smaller the probability of receiving an endowment lower than a given threshold. We denote the mean of \( y_t \) corresponding to \( F^0 \) and \( F^1 \) by \( \mu_0 \) and \( \mu_1 \), respectively, where \( \mu_0 < \mu_1 \) from FOSD. We assume that \( f^0(y_t) \) and \( f^1(y_t) \) are known to both the banker and the FDIC at the beginning of the initial period.

The bank can be terminated in any period \( \tau \). Upon termination, the banker receives utility \( J_\tau \geq 0 \) from the outside option, and the FDIC receives \( L_\tau \geq 0 \) units of the good.
from the liquidation of the long-term investment. For simplicity, we assume that \( J = J = 0 \) and \( L = 0, \forall \tau \). We also assume \( L < \max \{ \mu_1 - \psi, \mu_0 \} / (1 - \beta) \), which implies socially costly liquidation. There are no future interactions between the banker and the FDIC after \( \tau \). Thus \( Y_t = 0 \) for all \( t > \tau \).

We denote the history of realized endowments up to period \( t \) by \( y^t \equiv \{y_1, ..., y_t\} \). Let \( \lambda_t \) indicate whether the bank was terminated in period \( t - 1 \) (\( \lambda_t = 1 \)) or not (\( \lambda_t = 0 \)) for all \( t \geq 1 \). \( \lambda_t = 0 \) means that the bank is active at the beginning of period \( t \). Then we denote the set of all possible histories of endowments and termination up to \( t \) by \( H_t \equiv S^t \times \{0, 1\}^t \) and a history by \( h_t = (y^t, \lambda^t) \in H_t \). Also, we denote by \( \Lambda_t \) the set of histories of termination/survival where termination occurred at or before \( t - 1 \).

An allocation of resources in this environment is a stochastic vector process specifying consumption and termination \( (c, x, p) = \{c_t, x_t, p_t\}_{t=1}^\infty \), where \( c_t : H_t \to \mathbb{R}_+ \), \( x_t : H_t \to \mathbb{R} \), and \( p_t : H_t \to [0, 1] \). Here, \( c_t(h_t) \) is consumption by the banker after history \( h_t \), \( x_t(h_t) \) is consumption by the FDIC after history \( h_t \), and \( p_t(h_t) \) is the probability of termination by the FDIC after history \( h_t \). Note that we allow stochastic termination by the FDIC.

An allocation \( (c, x, p) \) is feasible if, \( \forall (t, h_t) \),

\[
\begin{align*}
&c_t(h_t) + x_t(h_t) \leq y_t, \\
&c_t(h_t) \geq 0, \\
&c_t(h_t) = x_t(h_t) = 0 \quad \text{if } \lambda^t \in \Lambda_t.
\end{align*}
\]

Beyond the physical restrictions, there are three additional contracting frictions in this environment. The first friction is that, at the beginning of period \( t \) the banker chooses \( e_t \), but the FDIC cannot observe it. The second is that, in the middle of period \( t \) the banker observes the realization of \( y_t \), but the FDIC does not. The third is that, at the end of period \( t \) after the banker receives his consumption \( c_t \), the banker can opt for termination.\(^{14}\)

Given any allocation, the banker can engage in three forms of deviations from the prescription of that allocation. First, the banker can choose the effort level different from the level the FDIC wants (henceforth the desirable level of effort). Second, the banker can pre-
tend to have a lower endowment realization than the actual in period $t$, that is, he can report $\hat{y}_t < y_t$. We assume that the banker cannot borrow, sell assets nor issue new equity, so that the report on endowments entails physical payment by the banker. Third, the banker can choose to terminate the bank depending on the realized history. He may terminate the bank, even if the FDIC does not. Also, in the initial period, the banker can choose to set up a bank or not.

A strategy $(e, \hat{y}, q) = \{e_t, \hat{y}_t, q_t\}_{t=1}^{\infty}$ is a stochastic vector process specifying the banker’s effort, report and termination decisions, where $e_t : H_t \rightarrow \{0, 1\}$, $\hat{y}_t : H_t \rightarrow S'(y_t) \equiv [y, y_t]$, and $q_t : H_t \rightarrow \{0, 1\}$. Here, $e_t(h_t)$ is the banker’s decision to exert high effort ($e_t = 1$) or low effort ($e_t = 0$) after history $h_t$, $\hat{y}_t(h_t)$ is the banker’s report on the realized value of $Y_t$ after history $h_t$, and $q_t(h_t)$ is the banker’s decision to terminate, or quit, the bank ($q_t = 1$) or not ($q_t = 0$) after history $h_t$. Note that $q_t(h_t) = 1$ if $q_{t-1}(h_{t-1}) = 1$, and that once the bank is terminated, the banker has no reason to exert high effort and the FDIC does not induce high effort from then on. We denote by $\Sigma$ the set of all possible strategies.

We define $W(c, x, p; e, \hat{y}, q)$ as the ex-ante continuation utility of the banker at the end of period 0, given an allocation $(c, x, p)$ and a strategy $(e, \hat{y}, q)$, that is,

$$W(c, x, p; e, \hat{y}, q) = E_0^{c} \left[ \sum_{t=1}^{\infty} \beta^t (Y_t - \hat{y}_t + c_t - \psi e_t) \right].$$

The expectation $E_0^{c}$ is associated with the distribution of $\{Y_t\}_{t=1}^{\infty}$ determined by $e$, and the termination time $\tau$ of the bank depends on $p$ and $q$.

Let $(e^*, \hat{y}^*, q^*) = \{e_t^*, \hat{y}_t^*, q_t^*\}_{t=1}^{\infty}$ denote the desirable-effort / truth-telling / no-quitting strategy, where $\hat{y}_t^*(h_t) = y_t$ and $q_t^*(h_t) = 0$ for all $(t, h_t)$, and $e_t^*(h_t) = 0$ if $\lambda' \in \Lambda_{t}$, and $e_t^*(h_t) = 0$ or 1 otherwise. An allocation $(c, x, p)$ is incentive-compatible if

$$W(c, x, p; e^*, \hat{y}^*, q^*) = \max_{(e, \hat{y}, q) \in \Sigma} W(c, x, p; e, \hat{y}, q).$$

An allocation that is both incentive-compatible and feasible is said to be incentive-feasible. Incentive-feasible allocations should induce the desirable level of effort, truth-telling of the realized income and no quitting by the banker in every period. The intuition
for constructing incentive-feasible allocations is as follows. First, in order to induce the desirable level of effort, a feasible allocation should possess a strong incentive, i.e., reward a good outcome and punish a bad outcome sufficiently. Second, using the Revelation Principle, we show that it is weakly optimal for the banker to tell the truth on the realized income, given a feasible allocation. Last, the banker terminates the bank in period \( t \), if the banker’s continuation utility derived from the allocation at the end of period \( t \) is less than the value of the outside option \( J_t \). Thus, in order to induce no quitting, \( p_t \) should be determined such that the FDIC terminates the bank with probability one if the banker is supposed to terminate the bank.

Given an incentive-feasible allocation \((c, x, p)\), the ex-ante continuation utility of the FDIC at the end of period 0 is given by

\[
V(c, x, p) = E_0^* \left[ \sum_{t=1}^{\infty} \delta^t x_t + \delta^t L \right],
\]

and the ex-ante continuation utility of the banker at the end of period 0 is given by

\[
W(c, x, p) = E_0^* \left[ \sum_{t=1}^{\infty} \beta^t (c_t - \psi e_t^*) \right],
\]

where \( \tau \) is the time the bank is terminated, which is determined by \( p \), and the expectation \( E_0^* \) is associated with \( f^* (y_t) \) for all \( t \).\(^{17}\)

The above environment is different from that of DeMarzo and Fishman (2002, henceforth, D-F) in the following two aspects. First, D-F model two sources of contracting frictions: private information and limited commitment by the agent. We model explicitly the banker’s choice of costly unobservable effort and the corresponding risk in addition to the two frictions. Thus, the banker’s continuation utility in our model is net of costs related to the desirable level of effort in every period before termination, while the agent’s continuation payoff in D-F is not associated with the costs of effort. Second, D-F consider both the case with a monopolistic agent and competing investors, and the other case with a monopolistic investor and competing agents. This paper focuses on the situation where the FDIC has the right
to charter a banker from a competitive pool, which is natural in the banking regulation setting. In particular, we emphasize the role of the initial capital requirement derived from the maximization problem of the FDIC.

### 3.2 Optimal Allocations

The goal of this subsection is to characterize the optimal incentive-feasible allocation in the above environment. Let \((c, x, p)\) be an incentive-feasible allocation, and \(\Gamma^*\) be the set of all incentive-feasible allocations. We set up the following ex-ante pseudo planner’s problem and derive the continuation function at the end of period 0:

\[
V(w) = \max_{(c, x, p) \in \Gamma^*} V(c, x, p) \quad s.t. \quad W(c, x, p) = w. \tag{1}
\]

An *optimal* allocation \((c^*, x^*, p^*)\) is a solution of (1). The continuation function \(V(\cdot)\) derived from (1) gives the highest possible continuation utility attainable by the FDIC, given a continuation utility \(w\) for the banker. Once we choose any optimal allocation, we fix a value of \(w\), which specifies a point on the continuation function. Note that \(V\) may have an increasing region.

From feasibility, we get \(x_t = y_t - c_t\). Thus, we redefine a feasible allocation as a pair \((c, p)\). Now we rewrite the ex-ante pseudo planner’s problem as the following sequence problem, \(PP(w)\).

\[PP(w): \text{Ex-ante pseudo planner’s problem}\]
\[ V(w) = \max_{\{c_t, p_t\}_{t=1}^{\infty}} E_0^* \left[ \sum_{t=1}^{\infty} \delta^t(Y_t - c_t) + \delta^t L \right] \]

\[ s.t. \ E_0^* \left[ \sum_{t=1}^{\infty} \beta^t(c_t - \psi e_t^*) \right] = w, \]

\[ (e^*, \hat{y}^*, q^*) \in \arg \max_{(e, \hat{y}, q)} E_0^* \left[ \sum_{t=1}^{\infty} \beta^t(Y_t - \hat{y}_t + c_t - \psi e_t) \right], \]

\[ c_t \geq 0, \ \forall t, h_t, \]

\[ 0 \leq p_t \leq 1, \ \forall t, h_t, \]

\[ Y_t = 0, \ \forall t > \tau, \]

\[ \hat{y}_t \leq y_t, \ \forall t, h_t. \]

Next, we define a recursive formulation for the pseudo planner’s problem in the following functional equation, \( FE \).

\( FE: \) Static pseudo planner’s problem

\[ v(w_0) = \max_{(c, p)} \delta E^* [y - c(y) + [1 - p(y)] v(w(y)) + p(y)L] \]

\[ s.t. \ \beta E^*[c(y) - \psi e^* + [1 - p(y)] w(y)] = w_0, \]

\[ E^*[c(y) - \psi e^* + [1 - p(y)] w(y)] \geq E^{-e^*}[c(y) - \psi(1 - e^*) + [1 - p(y)] w(y)], \]

\[ c(y) + [1 - p(y)] w(y) \geq y - y' + c(y') + [1 - p(y')] [1 - q'] w(y'), \ \forall y' \leq y, \ \forall q', \]

\[ c(y) \geq 0, \ \forall y, \]

\[ 0 \leq p(y) \leq 1, \ \forall y, \]

\[ w_{glb} \leq w(y) \leq w_{lub}, \]

where \( E^* \) is the expectation with respect to \( f^e \), \( E^{-e^*} \) is the expectation with respect to the complement of \( f^e \) (i.e., if \( e^* = 1 \), \( -e^* = 0 \), and vice versa), \( w_{glb} \) is the greatest lower bound and \( w_{lub} \) is the least upper bound for the value of \( w(y) \). In this problem, \( w_{glb} = J = 0.18 \)

Let \( (c^*, p^*) \) be an optimal allocation that satisfies the ex-ante pseudo planner’s problem, \( PP(w) \). Define \( w_t(h_t, \lambda_{t+1}) \) as the banker’s continuation utility at the end of period \( t \) after history \( h_t = (y^t, \lambda^t) \) and \( \lambda_{t+1} \), where

\[ w_t(h_t, \lambda_{t+1}) \equiv W_t(c^*, p^*; h_t, \lambda_{t+1}) = E_t^* \left[ \sum_{s=t+1}^{\infty} \beta^{s-t}(c_s(h_s | h_t, \lambda_{t+1}) - \psi e_s^*) \right]. \]
We can solve the ex-ante pseudo planner’s problem in a recursive manner. Instead of having the FDIC choose \((c_t, p_t)\) as a function of the history \(h_t = (y^t, \lambda^t)\), we let the FDIC choose the allocation \((c_t, p_t)\) as a function of \(w_{t-1}\) and \(y_t\), and choose the law of motion for \(w_t\) which specifies the continuation utility of the banker from period \(t + 1\) on as a function of \((w_{t-1}, y_t, \lambda_{t+1})\).

Before we fully characterize the continuation function \(v_t(\cdot)\) at the end of period \(t\) recursively, it is useful to consider the first-best continuation function. In the first-best setting, there is no hidden effort, no hidden income and no termination by the banker. Now, it is optimal for the FDIC to maximize the expected value of the sum of the discounted endowment flows and the discounted liquidation value, and then provide the banker’s continuation utility with a transfer payment. The first-best total continuation utility at the end of period \(t\) is calculated as

\[
V_{fb}^t = \max_{\tau^{fb} > t} E_t^e \left[ \sum_{s=t+1}^{\infty} \delta^{s-t} (Y_s - \psi e_t) + \delta^{\tau^{fb}-t} L \right],
\]

where \(\tau^{fb}\) is the time the bank is terminated by the FDIC in the first-best sense. Then, the first-best continuation function at the end of period \(t\) is given by \(v_{fb}^t(w) = V_{fb}^t(w)\). In period \(\tau^{fb}\) when the bank is terminated, \(V_{\tau^{fb}}^t = 0\), so \(v_{\tau^{fb}}^t(w) = -w\). Once the bank is terminated, no agency problem remains and the continuation function is linear.

From the assumption that liquidation is socially costly, \(V_{fb}^t > L\) holds, so that the first-best termination never happens and we can set \(\tau^{fb} = \infty\). Thus, if \((\mu_1 - \mu_0) > \psi\), \(V_{fb}^t = \delta(\mu_1 - \psi) / (1 - \delta)\), and if \((\mu_1 - \mu_0) < \psi\), \(V_{fb}^t = \delta \mu_0 / (1 - \delta)\). In general, the level of effort in each period in the first-best setting is set to maximize \(E[y_t | e_t] - \psi(e_t)\).

Once we consider incentive compatibility and the possibility of stochastic termination by the FDIC, the continuation function \(v_t(\cdot)\) is generally concave as shown below. The intuition is that, as the banker’s continuation utility decreases, it becomes difficult for the FDIC to punish the banker by lowering the banker’s continuation utility. Thus, as the banker’s continuation utility decreases, the FDIC’s continuation utility increases at a slower rate or even decreases. If the continuation function is not fully concave, we can use a randomization to concavify the function.
Suppose there is a concave continuation function \( v_t(\cdot) \), which gives the maximum value of the FDIC’s continuation utility, given a value of the banker’s continuation utility. Now we introduce consumption by the banker and termination by the FDIC. We know that if this can expand the continuation function, that is, increase the FDIC’s continuation utility further given the same value of the banker’s continuation utility, the FDIC will use these tools. In particular, if providing one unit of consumption right now to the banker is cheaper than promising one unit of continuation utility, then the FDIC will use consumption instead of continuation utility to reward the banker. Also, if termination of a bank gives higher continuation utility to the FDIC given a value of the banker’s continuation utility, the FDIC will terminate the bank.

Given \( v_t(\cdot) \), let \( \bar{w}_t = \inf \{ w | v_t'(w) \leq -1 \} \) be the minimum value of the banker’s continuation utility above which the FDIC has to sacrifice one or more units of the consumption to provide one more unit of the continuation utility to the banker. Denote by \( w_t \) the banker’s continuation utility at the point of tangency of the line constituting the convex hull of the continuation function \( v_t(\cdot) \) and the utility from termination \((0, L)\). Let \( l_t \) be the slope of this tangent line. Also, denote by \( \bar{w}_t^1 \) the threshold of the banker’s continuation utility, below which the FDIC wants to induce high effort with probability one, and by \( \bar{w}_t^0 \) the threshold, above which the FDIC wants to induce low effort with probability one. When \( f^e \) and \( L \) are given, \( \{ \bar{w}_t, w_t, \bar{w}_t^1, \bar{w}_t^0 \} \) is determined endogenously.

To show how the optimal allocation is determined, we define the following intermediate
state variables, \( w_t^c, w_t^{c1}, \) and \( w_t^{c0} \), which are functions of the state variables \( w_{t-1} \) and \( y_t \):

\[
\begin{align*}
  w_t^c &\equiv (\beta^{-1}w_{t-1} + \psi + y_t - \mu_1) \text{ when } \mu_1 - \mu_0 \geq \psi; \\
  w_t^{c1} &\equiv (\beta^{-1}w_{t-1} + \psi + \frac{\psi}{\mu_1 - \mu_0}(y_t - \mu_1)) \text{ when } \mu_1 - \mu_0 < \psi \text{ and } w_{t-1} \leq \overline{w}_{t-1}^{1}; \\
  w_t^{c0} &\equiv (\beta^{-1}w_{t-1} + y_t - \mu_0) \text{ when } \mu_1 - \mu_0 < \psi \text{ and } w_{t-1} \geq \overline{w}_{t-1}^{0}; \\
  \overline{w}_t^c &\equiv (\beta^{-1}\overline{w}_{t-1}^{1} + \psi + \frac{\psi}{\mu_1 - \mu_0}(y_t - \mu_1)); \\
  \overline{w}_t^{c0} &\equiv (\beta^{-1}\overline{w}_{t-1}^{0} + y_t - \mu_0).
\end{align*}
\]

Proposition 1 shows the concavity of \( v_t(\cdot) \) and states the optimal allocation \((c_t^*, p_t^*)\) and the law of motion of \( w_t \). The proofs of Propositions 1 and 2 and Corollary 1 are in the Appendix.

**Proposition 1** (1) If \( v_t(\cdot) \) is concave, \( v_{t-1}(\cdot) \) is also concave for all \( t \).

(2) When \( \mu_1 - \mu_0 \geq \psi \), the FDIC always induces high effort, and thus the optimal allocation and the law of motion for \( w_t \) are as follows:

\[
\begin{align*}
  c_t^*(w_{t-1}, y_t) &= \max \{w_t^c - \overline{w}_t, 0\}, \\
  p_t^*(w_{t-1}, y_t) &= \max \{0, \min \{1, (w_t^c - w_t^{c1})/\overline{w}_t\}\}, \\
  w_t &= \min \{\overline{w}_t, \max \{w_t, \beta^{-1}w_{t-1} + \psi + y_t - \mu_1\}\}.
\end{align*}
\]

(3) When \( \mu_1 - \mu_0 < \psi \), the optimal allocation and the law of motion for \( w_t \) depend on \( w_{t-1} \).

\[\text{\text{(i)} When } w_{t-1} \leq \overline{w}_{t-1}^{1}, \text{ the FDIC induces high effort at } t. \text{ Thus, }
\begin{align*}
  c_t^*(w_{t-1}, y_t) &= \max \{w_t^{c1} - \overline{w}_t, 0\}, \\
  p_t^*(w_{t-1}, y_t) &= \max \{0, \min \{1, (w_t^{c1} - w_t^{c})/\overline{w}_t\}\}, \\
  w_t &= \min \{\overline{w}_t, \max \{w_t^{c1}, \beta^{-1}w_{t-1} + \psi + \frac{\psi}{\mu_1 - \mu_0}(y_t - \mu_1)\}\}\}.\]
\]

\[\text{\text{(ii)} When } w_{t-1} \geq \overline{w}_{t-1}^{0}, \text{ the FDIC induces low effort at } t. \text{ Thus, }
\begin{align*}
  c_t^*(w_{t-1}, y_t) &= \max \{w_t^{c0} - \overline{w}_t, 0\}, \\
  p_t^*(w_{t-1}, y_t) &= \max \{0, \min \{1, (w_t^{c0} - w_t^{c})/\overline{w}_t\}\}, \\
  w_t &= \min \{\overline{w}_t, \max \{w_t^{c0}, \beta^{-1}w_{t-1} + y_t - \mu_0\}\}\}.\]
\]
(iii) When $w_{t-1}^1 < w_{t-1} < w_{t-1}^0$,

with probability $p_t^* = \left(\frac{w_{t-1}^0 - w_{t-1}}{w_{t-1}^0 - w_{t-1}^1}\right)$, the FDIC induces high effort at $t$, and

$$c_t^*(w_{t-1}, y_t) = \max \left\{w_{t-1}^0 - \bar{w}_t, 0\right\},$$

$$p_t^*(w_{t-1}, y_t) = \max \left\{0, \min\left\{1, \frac{(w_t - w_{t-1}^0)}{w_{t-1}}\right\}\right\},$$

$$w_t = \min \left\{\bar{w}_t, \max \left\{w_t, \beta^{-1}w_{t-1}^1 + \psi + \frac{\psi}{\mu_1 - \mu_0}(y_t - \mu_1)\right\}\right\};$$

with probability $1 - p_t^*$, the FDIC induces low effort at $t$, and

$$c_t^*(w_{t-1}, y_t) = \max \left\{w_{t-1}^0 - \bar{w}_t, 0\right\},$$

$$p_t^*(w_{t-1}, y_t) = \max \left\{0, \min\left\{1, \frac{(w_t - w_{t-1}^0)}{w_{t-1}}\right\}\right\},$$

$$w_t = \min \left\{\bar{w}_t, \max \left\{w_t, \beta^{-1}w_{t-1}^0 + y_t - \mu_0\right\}\right\}.$$

Note that $w_{t-1}^c$ is important in determining the optimal allocation: $c_t(w_{t-1}^c) = \max(w_{t-1}^c - \bar{w}_t, 0)$ and $p_t(w_{t-1}^c) = \frac{(w_{t-1}^c - w_{t-1})}{w_{t-1}}$. When $w_{t-1}^c$ is high enough, the banker can enjoy positive consumption, while when $w_{t-1}^c$ is low, the FDIC terminates the bank stochastically. Figures 1 and 2 illustrate how the optimal allocation is determined as a function of $w_{t-1}^c$ given $v_t^c(\cdot)$.

The proof of Proposition 1 basically follows the structure of D-F. The difference is that hidden choice of risk by the banker is explicitly added in the agency stage. The first result of Proposition 1 comes from the multi-stage structure of D-F. D-F also show that, in models with binary (high/low) hidden effort choice, if the first-best level of effort is high effort, then the optimal contract takes the same form as that without hidden effort. This corresponds to the second result of Proposition 1. Thus, the additional incentive compatibility constraint associated with hidden effort does not bind. On the other hand, the third result shows that, when $\mu_1 - \mu_0 < \psi$, the optimal allocation can differ depending on the level of the banker’s continuation utility. It is possible that, when the continuation utility is relatively high, the FDIC wants to induce low effort, but that, when the continuation utility is relatively low, it wants to induce high effort.

The intuition for this result is as follows. Suppose that high effort yields the realized income that is higher in the sense of FOSD but still $\mu_1 - \mu_0 < \psi$ holds, and that high effort also lowers the variance of the realized income. Then, when the banker’s continuation utility
is relatively high, the continuation function is almost linear. Thus, the benefit from reducing variance by inducing high effort is small, and saving the cost of high effort \( \mu_0 - (\mu_1 - \psi) \) is more important. On the other hand, when the banker’s continuation utility is relatively low, the continuation function is highly concave. Now the benefit from reducing variance by inducing high effort is larger than that from saving the cost of high effort.\(^{20}\)

Finally, we consider the optimal choice of the initial condition \( w_0 \) and the initial capital requirement \( K_0 \). The initial period has two stages. In the first stage, as a monopolistic regulator, the FDIC proposes a dynamic allocation to a potential banker from a competitive pool.\(^{21}\) The FDIC offers the required level of initial transfer from the banker to the FDIC, \( K_0 \). If the initial wealth of a potential banker \( \varepsilon_0 \) is less than \( K_0 \), he cannot accept the offer. If \( \varepsilon_0 \) is equal to or greater than the required initial transfer \( K_0 \), he will accept the offer, as long as he breaks even, i.e. \( K_0 \leq w_0^c \), where \( w_0^c \) is the continuation utility of the banker right before consumption by the banker in period 0, and \( w_0^c = c_0 + w_0 \). Using backwards induction, we can derive the continuation function before consumption in period 0, \( v_0^c(\cdot) \).

In the second stage of period 0, the banker consumes \( c_0 \) and the FDIC exercises stochastic termination \( p_0 \), as it does in period \( t \geq 1 \). As is clear in the proof of Corollary 1, consumption \( c_0 \) by the banker in period 0 is essentially a partial refund of the initial transfer \( K_0 \). Thus we can ignore the second stage of the initial period. Corollary 1 shows that the optimal choice of \( K_0 \) and \( w_0 \) coincides with \( w_0 \).

**Corollary 1** \( K_0^* = w_0^* = w_0 \).

### 4 Implementation

In Section 3, we considered the optimal allocation in a dynamic setting. This section shows that the ex-ante optimal allocation can be implemented by an appropriate combination of a book-value capital regulation and a risk-based deposit insurance premium.

We now consider the environment of Section 3 in a banking context, in which the FDIC
first commits to a set of banking regulation and then the banker chooses his action in every period. \( \{ Y_t \}_{t=1}^{\infty} \) represents independent cash flows or profit streams from the assets of the banker. The FDIC and the banker have the same risk-neutral preferences as before. The banker can exert costly unobservable effort, which affects the distribution of \( Y_t \) as before. The banker can observe the realization of \( Y_t \), but the FDIC cannot. Thus the FDIC relies on the banker’s report of \( y_t \), which we again denote by \( \hat{y}_t \). Also, the banker can quit anytime in period \( t \). We denote by \( \lambda^t \) the termination history up to \( t - 1 \).

The timing of events is as follows. At the beginning of the initial period, both the FDIC and a potential banker know the distributions of \( Y_t \), \( f^0 \) and \( f^1 \). The FDIC makes the banker a take-it-or-leave-it offer, which consists of the initial required capital \( K_0 \), the deposit insurance premium \( \bar{x}_t \), the dividend payment \( d_t \), and the termination probability \( p_t \) in every period, where \( (\bar{x}_t, d_t, p_t) \) are functions of the level of book-value capital. If the banker accepts the offer, he pays \( K_0 \) and opens the bank. Otherwise, he rejects the offer and enjoys the outside option \( 0 \). Once a bank is set up, the banker receives 1 unit of deposits which are invested in the project. At the end of period 0, the level of book-value capital is \( K_0 \). We assume that only deposits are invested; the initial capital is kept as cash to meet possible future liquidity needs, such as paying a deposit insurance premium. Alternatively, we can assume that the capital grows at a risk-free rate \( r_f \). But as long as \( r_f < 1/\beta - 1 \), the qualitative results do not change.

The banker starts period 1 with capital \( K_0 \). First, the banker chooses \( e_1, y_1 \) is realized, and then the banker reports \( \hat{y}_1 \) to the FDIC, or equivalently, adds \( \hat{y}_1 \) to \( K_0 \). Based on \( K_0 \) and \( \hat{y}_1 \), the FDIC charges \( \bar{x}_1 \). The new level of capital becomes \( K_1^d = K_0 + \hat{y}_1 - \bar{x}_1 \). Then, the FDIC allows the dividend payment \( d_1 \), and the banker consumes \( y_1 - \hat{y}_1 + d_1 \). The dividend is publicly observable consumption, while \( (y_t - \hat{y}_t) \) is private consumption. If \( K_1^d < K_1 \), the FDIC either terminates the bank with probability \( p_1 \), or bails out the bank by recapitalizing it, so that \( K_1 = K_1^d \) with probability \( 1 - p_1 \). If the bank is not terminated by the FDIC, the banker can choose either to terminate the bank \( (q_1 = 1) \) or to continue into period 2.
\(q_1 = 0\). If the bank is not terminated by the end of period 1, the same events repeat in period 2. Figure 3 summarizes the timing of events.

In the context of banking regulation, the FDIC commits to the following standard regulation at the beginning of the initial period:

**Deposit Insurance Premium:** A deposit insurance premium is characterized by a sequence of payments \(\{\bar{x}_t\}\) from the banker to the FDIC. If a premium is not paid to the FDIC, the bank is undercapitalized.

**Book-Value Capital Regulation:** A book-value capital regulation is characterized by (i) the initial capital infusion \(K_0\), (ii) the dividend payment if the current level of capital \(K^d_t\) is above an upper bound \(\overline{K}_t\), and (iii) being undercapitalized if \(K^d_t\) is below a lower bound \(\underline{K}_t\).

**Undercapitalization and Stochastic Termination/Bailout:** If \(z > 0\) is the amount of undercapitalization in period \(t\), the FDIC liquidates the bank and keeps \(L_t\) with probability \(p_t(z) = z/K_t\), or bails out the bank by increasing the level of capital by \(z\), so that the level of capital becomes \(K'_t\) with probability \(1 - p_t(z)\).

Given this regulation, the banker optimally chooses his strategy \(\{e_t, \hat{y}_t, q_t\}_{t=1}^{\infty}\). In particular, the banker chooses in every period whether to exert the desirable level of effort \((e_t = e^*_t)\) or not \((e_t \neq e^*_t)\), whether to report truthfully \((\hat{y}_t = y_t)\) or consume privately \((\hat{y}_t < y_t)\), and whether to terminate the bank \((q_t = 1)\) or not \((q_t = 0)\).

We show that the above capital regulation and deposit insurance premium can implement the optimal allocation in two steps. First, we show that a combination of the above regulatory instruments generates the outcome equivalent to the ex-ante optimal allocation, assuming that the banker exerts the desirable level of effort, never quits, and chooses to use all of the realized return to increase book-value capital in every period (i.e., the banker chooses to enjoy no private consumption). Second, we show that the banker who wants to maximize \(E_t^s \left[ \sum_{s=t}^{\infty} \beta^{s-t}(Y_s - \hat{y}_s + d_s - \psi e_t) \right]\) finds the desirable-effort / truth-telling / no-quitting strategy optimal, for all \(t\) and after any history \((\hat{y}^{t-1}, \lambda^t)\) summarized by \(K_t\). Proposition 2
shows the exact forms of deposit insurance premiums and capital regulation.

**Proposition 2** The ex-ante optimal allocation is equivalent to the outcome of the following combination of a book-value capital regulation and a risk-based deposit insurance premium.

1) Suppose $\mu_1 - \mu_0 \geq \psi$. Then, the deposit insurance premium is $\bar{x}_t = \mu_1 - \psi - K_{t-1}(\beta^{-1} - 1)$, which is decreasing in the level of book-value capital at the beginning of period $t$, $K_{t-1}$. The book-value capital regulation consists of the initial required capital $K_0 = \overline{K}_0$, the dividend payment $d_t = \max(K_t^d - \overline{K}_t, 0)$ and the termination probability $p_t = \max[(K_t - K_t^d)/K_t, 0]$, where $\overline{K}_t = \overline{w}_t$ and $K_t = w_t$. The law of motion for $K_t$ is $K_t = \min\{K_t, \max\{K_t, K_{t-1} + y_t - \bar{x}_t\}\}$.

2) Suppose $\mu_1 - \mu_0 < \psi$. Then, (i) when $K_{t-1} < \overline{K}_t^1$, the deposit insurance premium is $\bar{x}_t^1 = (\psi/(\mu_1 - \mu_0))\mu_0 - K_{t-1}(\beta^{-1} - 1) - y_t(\psi/(\mu_1 - \mu_0) - 1)$, which is decreasing in both the level of book-value capital at the beginning of period $t$, $K_{t-1}$, and the realized return, $y_t$; (ii) when $K_{t-1} > \overline{K}_t^0$, the deposit insurance premium is $\bar{x}_t^1 = \mu_0 - K_{t-1}(\beta^{-1} - 1)$, which is decreasing in $K_{t-1}$; (iii) when $\overline{K}_t^1 \leq K_{t-1} \leq \overline{K}_t^0$, either with probability $\hat{p}_t = (K_t^0 - K_{t-1})/(\overline{K}_t^0 - \overline{K}_t^1)$, the FDIC charges $\left[\beta^{-1}(K_{t-1} - \overline{K}_t^1)\right]$ to the banker and then the banker pays the deposit insurance premium $\bar{x}_t = (\psi/(\mu_1 - \mu_0))\mu_0 - K_{t-1}(\beta^{-1} - 1) - y_t(\psi/(\mu_1 - \mu_0) - 1)$, or with probability $1 - \hat{p}_t$, the FDIC pays $\left[\beta^{-1}(K_t^0 - K_{t-1})\right]$ to the banker and then the banker pays the deposit insurance premium $\bar{x}_t^1 = \left[\mu_0 - K_{t-1}(\beta^{-1} - 1)\right]$. The book-value capital regulation consists of the initial required capital $K_0 = \overline{K}_0$, the dividend payment $d_t = \max(K_t^d - \overline{K}_t, 0)$, the termination probability $p_t = \max[(K_t - K_t^d)/K_t, 0]$, and the randomization $\hat{p}_t$ over induced effort, where $\overline{K}_t = \overline{w}_t$, $K_t = w_t$, $K_t^0 = w_{t-1}$, and $\overline{K}_t^1 = \overline{w}_{t-1}$. For (i) and (ii), the law of motion for $K_t$ is $K_t = \min\{K_t, \max\{K_t, K_{t-1} + y_t - \bar{x}_t^1\}\}$. For (iii), the law of motion for $K_t$ is $K_t = \min\{K_t, \max\{K_t, K_{t-1} + y_t - \left[\beta^{-1}(K_{t-1} - \overline{K}_t^1)\right] - \bar{x}_t^1\}\}$ with probability $\hat{p}_t$, or $K_t = \min\{K_t, \max\{K_t, K_{t-1} + y_t + \left[\beta^{-1}(K_t^0 - K_{t-1})\right] - \bar{x}_t^1\}\}$ with probability $1 - \hat{p}_t$.

The dividend and termination rules in this implementation are the same as the consumption and termination rules in the ex-ante optimal allocation. Note that the level of capital $K$ replicates the law of motion of $w$, so now $K$ works as a record-keeping device. Also note
that stochastic termination is coupled with stochastic bailout. If $K_t > K^d_t$, the bank is either terminated with probability $p_t$ or bailed out with probability $1 - p_t$.\footnote{23}

The risk-based deposit insurance premiums in Proposition 2 take three different forms. First, when $\mu_1 - \mu_0 \geq \psi$, the deposit insurance premium in period $t$ is decreasing only in the capital level at the beginning of period $t$. When $\mu_1 - \mu_0 < \psi$ and $K_{t-1} < K^0_t$, the deposit insurance premium in period $t$ is decreasing in both the capital level at the beginning of period $t$ and the return of the current period. The intuition for this difference is as follows. When $\mu_1 - \mu_0 < \psi$ and $K_{t-1} < K^0_t$, the optimal allocation prescribes that the banker be rewarded with the increase in the continuation utility by more than one dollar, given an increase in profit by one dollar. To implement this allocation, the banker is rewarded with more than one dollar, through the increase in the dividend by one dollar, as well as through the decrease in the deposit insurance premium, given an increase in the profit by one dollar. Thus, when inducing the positive effort is more costly, the deposit insurance premium should be more strongly risk-based, while the book-value capital regulation remains the same. Finally, when $\mu_1 - \mu_0 < \psi$ and $K_{t-1} > K^0_t$, the deposit insurance premium in period $t$ is decreasing only in the capital level at the beginning of period $t$. The intuition is that since the capital level is high enough, the FDIC wants to induce less costly effort, i.e., low effort, and thus the deposit insurance premium does not need to be strongly risk-based.

It should be noted that, under a set of parameters and probability distributions, the FDIC may want to randomize over induced effort. This requires the FDIC to adjust the level of capital up or down depending on the result of the randomization $\tilde{p}_t$. Instead of incorporating these adjustments in the deposit insurance premium, we separate them from the deposit insurance premium: the adjustments are paid before the level of effort is chosen, whereas the deposit insurance premium is paid after it is chosen.

D-F show that a set of simple financial contracts can implement an optimal long-term financial contract, while we show that the well-designed combination of a risk-based deposit insurance premium and a bank capital regulation can implement the ex-ante optimal dynamic
allocation. In particular, D-F use credit line balance as the record-keeping device: the amount of credit line balance determines in every period if the investor allows dividends or terminates the project. In this paper, we use the level of book-value capital as the record keeping device.\textsuperscript{24} Also, D-F use a long-term debt as a tool to coordinate the level of credit line with the agent’s continuation utility, while this paper uses a risk-adjusted deposit insurance premium to coordinate the level of book-value capital with the banker’s continuation utility. Both the long-term debt in D-F and the deposit insurance premium in this paper work as income transfer from the agent to the principal. However, they differ in that in a stationary setting the coupons of the long-term debt are constant over time, while the deposit insurance premium depends on the level of bank capital as long as bank capital grows at a risk-free rate $r_f < 1/\beta - 1$ or as long as $\mu_1 - \mu_0 < \psi$ and $K_{t-1} < K^1_t$.

So far, we have considered a model in which the size of the bank’s assets is fixed over time and the whole bank is subject to stochastic termination. In reality, it is possible that a regulator has difficulty in stipulating a stochastic termination rule in a law or convincing the stakeholders of a bank that he credibly implements stochastic termination. Also, it is frequently observed in practice that a bank adjusts the size of its assets to meet a required capital ratio. Therefore, it is important to see if partial termination can be used instead of stochastic termination to implement the optimal allocation. We can show that stochastic termination and partial termination are equivalent\textsuperscript{25} under the assumption that partial termination of the bank scales down all the cash flows including the costs associated with effort.\textsuperscript{26}

5 Comparative Statics

5.1 Liquidation Value and Capital Requirements

In Section 4, we showed that the initial capital requirement $K_0$, the dividend threshold $K_t$ and the termination threshold $K_t$ depended on the shape of the continuation function. Given
that the continuation function is stationary, \( K_t = \bar{K} \) and \( K_t = \bar{K} \) for all \( t \geq 0 \). In this subsection, we investigate how the continuation function changes its shape as the liquidation value \( L \) changes. In particular, we are interested in the correlation between the liquidation value and the optimal capital requirements, \( \bar{K} \) and \( \bar{K} \). We define \( w^M = \inf \{ w : v'(w) \leq 0 \} \). Then, \( w^M \) maximizes \( v \), so that \( v(w) < v(w^M) \) for \( w < w^M \). We also define \( L^m \) as the value of \( L \) such that \( v(w^M; L) = L \). Depending on the value of \( L \), the continuation function takes on three different shapes.

(Case 1) Suppose the liquidation value \( L \) is equal to or greater than the maximum total continuation utility. That is, the recovery value is close to the first-best value of the assets. Then, \( l \leq -1 \) and \( w = \infty \). Thus, it is optimal to terminate the bank with probability one for any value of \( K_t \), and the continuation function is linear. This case shows that it is crucial to have costly liquidation in order to have a strictly concave continuation function.

(Case 2) Suppose \( L \) is equal to or greater than \( L^m \) but less than the maximum total continuation utility. This is the case when the recovery value is relatively high. Then, \( -1 < l_t \leq 0 \) and \( w^M \leq w < \bar{w} \). Thus, if the bank’s capital \( K^d_t \) is above \( K = w \), the bank is not terminated. Otherwise, the bank faces stochastic termination by the FDIC. Since \( w^M \leq w \), the optimal allocation is on the Pareto frontier, and \( v(\cdot) \) is decreasing and concave in \( w \in [w, \bar{w}] \).

(Case 3) Suppose \( L \) is less than \( L^m \). Then, \( l_t > 0 \) and \( w^M > w \). Again, if \( K^d_t \geq K = w \), the bank is not terminated. Otherwise, the bank faces stochastic termination. Since \( w^M > w \), the continuation function at the end of period \( t \) now has an increasing region. If the banker’s continuation utility is between \( w \) and \( w^M \), this is Pareto-inferior because both the banker and the FDIC would like to replace it with a new, Pareto-improving allocation. However, since we assume that renegotiation is impossible and that the FDIC can commit to the allocation, the continuation function is a pseudo-Pareto frontier. This inefficient region is important, because the FDIC might use this low continuation utility ex post to provide an
incentive/threat to the banker.

To illustrate the above three cases and show the relationship between the liquidation value and the capital thresholds, we calibrate the model using the following parameters:

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\underline{\psi} - \psi$</th>
<th>$\bar{\psi} - \psi$</th>
<th>$\mu_1 - \psi$</th>
<th>$J$</th>
<th>$D$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.95</td>
<td>-0.005</td>
<td>0.015</td>
<td>0.005</td>
<td>0</td>
<td>1</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

The annual returns on assets of all FDIC-insured thrift institutions between 1984 and 2002 have the mean 0.449%, the standard deviation 0.515%, the maximum 1.122%, and the minimum -0.391%. Thus, for computational simplicity, we choose $\mu_1 - \psi = 0.5\%$, $\sigma = 0.5\%$, $\underline{\psi} - \psi = -0.5\%$, and $\bar{\psi} - \psi = 1.5\%$. Then, we simulate the return distribution after subtracting the cost of effort by the truncated normal distribution with the mean 0.005, the standard deviation 0.005 and the support $[-0.005, 0.015]$. Note that we assume $\mu_1 - \psi > \mu_0$ in this calibration. Figure 4 shows an example for each case. In Figure 4, the unit is basis point, so that 100 means 1%.

Note that the termination threshold is the same for each case. In particular, the termination threshold is determined as $\beta(\mu_1 - \psi - \underline{\psi})$. Thus, as long as the means and the lower bounds of the return distributions of two banks are the same, the termination thresholds for both banks should be the same. On the other hand, if the return distributions of two banks have the same mean and variance but different supports, then the termination thresholds are different from each other. The annual returns on assets of all FDIC-insured thrift institutions between 1984 and 2002 have the mean 0.449%, the standard deviation 0.515%, the maximum 1.122%, and the minimum -0.391%. Thus, for computational simplicity, we choose $\mu_1 - \psi = 0.5\%$, $\sigma = 0.5\%$, $\underline{\psi} - \psi = -0.5\%$, and $\bar{\psi} - \psi = 1.5\%$. Then, we simulate the return distribution after subtracting the cost of effort by the truncated normal distribution with the mean 0.005, the standard deviation 0.005 and the support $[-0.005, 0.015]$. Note that we assume $\mu_1 - \psi > \mu_0$ in this calibration. Figure 4 shows an example for each case. In Figure 4, the unit is basis point, so that 100 means 1%.

Figure 5 shows the negative correlation of the liquidation value and the dividend threshold. Since the dividend threshold is also the optimal level of initial required capital, this result implies that, other things being equal, the FDIC requires a higher level of initial capital for a bank with a lower liquidation value. The intuition for this result is as follows. When the liquidation value is lower, the FDIC’s continuation utility decreases faster, as the banker’s continuation utility goes down and approaches the outside option value. Thus, the
continuation function becomes lower around the dividend threshold, and the threshold for dividend payment is higher for the bank with a lower liquidation value.

5.2 Mean-Preserving Spreads and Capital Thresholds

In this subsection, we investigate whether a higher level of risk in the form of mean-preserving spreads affects the optimal capital requirements. Suppose there are two banks with the same size $D$ of the long-term assets, the same liquidation value $L$, the same value of $\psi$ and the return distributions with the same mean but different variances. We denote the probability density functions of bank 1 and bank 2 by $f_1^e$ and $f_2^e$, respectively, and the standard deviations associated with $f_1^e$ and $f_2^e$ by $\sigma_1^e$ and $\sigma_2^e$ respectively, where $\sigma_1^e < \sigma_2^e$, $\forall e$. Thus, bank 2 is riskier than bank 1 in the sense of second-order stochastic dominance (SOSD) or a mean-preserving spread. Then, the natural question is how to optimally set risk-based deposit insurance premiums and bank capital regulation for the two different banks. Specifically, we ask if it is optimal to impose a stricter capital regulation to the riskier bank, to charge a higher deposit insurance premium for the riskier bank or to do both.

In the framework of the traditional option-value approach to deposit insurance, higher risk entails a higher deposit insurance premium, since the deposit insurance payment is identical to a put option to a bank’s assets. The option value increases as the riskiness of a bank increases. However, the deposit insurance premium in this paper is not calculated from a fair-pricing consideration, but from the consideration of resolving the incentive problems in the dynamic setting and inducing the ex-ante optimal allocations. In particular, the optimal deposit insurance premium for both banks, $\bar{x}_1$ and $\bar{x}_2$, are of the same form: when $\mu_1 - \mu_0 \geq \psi$, $\bar{x}_{1t} = \bar{x}_{2t} = \mu_1 - \psi - K_{t-1}(\beta^{-1} - 1)$; when $\mu_1 - \mu_0 < \psi$ and $K_{t-1} < K_t^1$, $\bar{x}'_{1t} = \bar{x}'_{2t} = (\psi/(\mu_1 - \mu_0)) \mu_0 - K_{t-1}(\beta^{-1} - 1) - y_t(\psi/(\mu_1 - \mu_0) - 1)$, etc. Thus, if bank 1 and bank 2 have the same levels of book-value capital and current-period profit, both will pay the same amount of deposit insurance premiums.

By contrast, the shape of the continuation functions for both banks is different. When
the liquidation value of the assets for both banks is the same, the optimal capital regulation prescribes that bank 2 should have the higher dividend threshold or the initial required capital, i.e., $K_1 < K_2$. To show how we get this result, we again assume that $\mu_1 - \mu_0 \geq \psi$ holds. We denote by $\{v_1, v_1^c, v_1^y\}$ the stationary continuation functions of bank 1 under $f_1^1$, and by $\{v_2, v_2^c, v_2^y\}$ those of bank 2 under $f_2^1$. We also denote by $\tilde{v}_1^y$ the continuation function derived from $v_1^c$ using the expectation over $f_2^1$. Then, from the property of SOSD and the fact that $v_1$ is concave, $v_1^y(w) \geq \tilde{v}_1^y(w)$ holds for all $w$, and $v_1^y(w) > \tilde{v}_1^y(w)$ for some $w$. By the Contraction Mapping Theorem, we have $v_1(w) \geq v_2(w)$ for all $w$, and $v_1(w) > v_2(w)$ for some $w$. In addition, $w_1 < w_2$. This is because $v_1(w) > v_2(w)$ but $\tilde{w}_1 = \tilde{w}_2 = \tilde{w}$ where $\tilde{w}_i = \inf \{w \mid v_i^c(w) \leq -\delta/\beta\}$, so that the slope of $v_2$ increases faster than that of $v_1$ as $w$ decreases from $\tilde{w}$.

Figure 6 illustrates an example of the mean-preserving spread. The dotted line stands for the continuation function under the truncated normal distribution (the return distribution of bank 1), and the solid line for the continuation function under the uniform distribution with the same mean and support (the return distribution of bank 2). In this example, the initial capital requirement of bank 1 is 2.4% while that of bank 2 is 2.95%.

In summary, an observed mean-preserving spread only affects the curvature or shape of the continuation function, not the deposit insurance premium schedule. Therefore, it is optimal to impose a stricter capital regulation to the riskier bank in the sense of SOSD, but to maintain the same deposit insurance premium schedule for both banks.

6 Discussion

In this section, we discuss the implications of the main results on selected regulatory policy issues. We first compare the US bank regulation with the model-implied regulation, and then look into the issue of regulatory forbearance in the model.
6.1 Model-Implied Regulation vs. Current Regulation

The deposit insurance premium implied by the model resembles that observed in practice. Currently in the United States, the risk-based deposit insurance premium depends on capital ratios and a CAMELS rating. In the model, the risk-based deposit insurance premium depends on the level of book-value capital and, under some circumstances, the current-period profit.

The regulatory scheme in the model implements the ex-ante optimal allocation without the possibility of renegotiation. This ex-ante commitment is largely consistent with the core idea of the current US PCA, specifying ex ante all contingent action rules of the regulators by law and allowing a minimal amount of regulatory discretion. The current US PCA and the model-implied capital regulation have in particular the following similarities and differences.

First, PCA essentially prohibits dividend payments, once a bank is classified as undercapitalized and likely to be terminated. On the other hand, the model-implied capital regulation has a range of capital levels where dividend payments are not allowed, even though the probability of termination is zero. Considering that undercapitalized banks are rarely terminated, that significantly undercapitalized banks are more likely terminated and that critically undercapitalized banks are almost certainly terminated, the model-implied capital regulation is similar to the current PCA. Both have a region of no (or rare) termination and no dividend payment. \( K_t \) and \( K_* \) in the model correspond respectively to the upper bound and the lower bound of capital ratios for the undercapitalized category.

Second, the model-implied capital regulation prescribes stochastic termination by the FDIC, where the probability of termination increases proportionately as the level of capital decreases below the threshold \( K_* \). On the other hand, the current PCA prescribes gradual intervention rules depending on the level of capital, but only allows deterministic termination. It should be noted here that the optimality of stochastic termination over deterministic termination in the model is driven by the discrete-time nature of the model and limited commitment by the banker. In a similar setting, DeMarzo and Sannikov (2006) show that
an optimal contract in continuous time does not require stochastic termination. In reality, a
bank’s book-value capital changes frequently, while the bank does not report the value to the
regulator frequently. If reporting becomes more frequent for an undercapitalized bank, there
will be less room for effective stochastic termination. Also, the current PCA prescribes more
intense interventions, as a bank becomes more deeply undercapitalized. These corrective
actions, which are not modeled in this paper, may be able to create incentives for the
banker to act in the FDIC’s interest when the bank is close to termination, just as stochastic
termination is able to do so. Moreover, it should be noted that in practice the implementation
of the rules tends to incorporate a stochastic element. Therefore, even though the model
prescribes stochastic termination, this feature of the contract may not be directly applicable
to evaluating PCA.29

Third, the idea of providing a subsidy to raise the level of capital up to the lower bound
once a bank survives stochastic termination is not in line with the current PCA. In PCA,
recapitalization by the banker such as issuing new equity is allowed to raise the level of capital
up to the lower bound and, if not recapitalized promptly, the bank is closed with probability
one. Thus in principle the current PCA does not allow bailouts by the government. On the
other hand, stochastic termination in this paper entails a stochastic bailout or subsidy.30

There are many papers in favor of “constructive ambiguity” of government policy. Freixas
(2000) shows that the optimal bailout policy of the lender of last resort can be a mixed
strategy under some circumstances. He interprets this result as confirmation of constructive
ambiguity. He argues that this result is in line with the central bankers’ claim that it
is efficient for them to have discretion in lending to individual institutions. Mishkin (1999)
prescribes constructive ambiguity in terms of dealing with a moral hazard problem, namely a
“too-big-to-fail” policy toward large financial institutions. He uses this term to provide room
for judgment by supervisors. He argues that, since the FDICIA allows regulators to use the
systemic risk exception when choosing a closure method, constructive ambiguity mitigates
the moral hazard problem. Note that he means by constructive ambiguity a contingent rule,
not a mixed strategy. By way of contrast, the stochastic termination rule in this paper is a set of mixed strategies: the probability of termination increases and the probability of bailout decreases, as the capital level decreases below a threshold. Therefore the stochastic termination/bailout rule is not a form of discretion, but a form of “structured ambiguity,” which is stronger than constructive ambiguity.

Finally, in this paper when a bank is terminated, the banker who is the owner and manager of the bank becomes separated from the bank and enjoys his outside option, while the bank’s assets are liquidated. In reality, most bank closures involve the FDIC using the option of deposit assumption rather than liquidation in order to minimize liquidation costs. Under deposit assumption, the FDIC can choose whether to get rid of the incumbent bank managers or keep them in the new bank. This requires the FDIC to sort out competent from incompetent managers. Modeling hidden types of managers is left for future work.

6.2 Ex-post and Ex-ante Forbearance

Regulatory forbearance is typically defined as a regulator’s tolerance of a bank operating with a very low or negative level of capital. In this paper, if a bank becomes undercapitalized and then recapitalized by the FDIC after surviving stochastic termination, the bank continues to stay in business. This is a form of forbearance in the ex-post sense. It is even possible that a bank with consecutive bad performances is recapitalized repeatedly for several periods, if the bank survives successive stochastic termination. However, despite ex-post forbearance, the optimal termination policy itself is prompt in the ex-ante sense. When a bank’s book-value capital falls below a threshold, the bank does go through stochastic termination by the FDIC. One way to understand this more clearly is to consider the equivalent partial termination which is both prompt and deterministic.

In the model, we can also consider ex-ante regulatory forbearance. In Section 3, we assumed that the FDIC was a monopolistic regulator who imposed a high level of the initial capital requirement. Suppose, on the contrary, that the FDIC is “captured” by the banking
industry, in the sense that the banker enjoys a positive rent just by being chartered to open a bank. Now the initial capital requirement is very low, and the FDIC is likely to exert ex-ante forbearance over time.

As an example, suppose the banker initially pays \( K_0^* = w_0 \ll \bar{w}_0 \) as the initial required capital, and the FDIC promises \( w_0 = \bar{w}_0 \) to the bank as continuation utility. The banker is likely to enjoy dividends from period 1 on, while the level of book-value capital is low. Now there is a discrepancy between \( K_0 \) and \( w_0 \). The optimal allocation in the stationary setting should preserve this difference \( (\bar{w}_0 - w_0) \) over time so that the deposit insurance premium is set to satisfy \( w_1^d = \frac{w_0^1 - K_0^*}{K_0} \). The level of capital imitates the dynamics of the banker’s continuation utility, but with a constant gap. Then, consecutive bad performances of the banker together with the deposit insurance premium will lower the level of capital. However, even when the bank’s capital is exhausted, the banker is not subject to stochastic termination until the level of capital drops to a very low level.

The US experience with savings and loan associations (S&Ls) in the 1980s may correspond to this case. In the 1980s, the regulator in charge of S&Ls amended accounting rules to allow S&Ls to use overly optimistic franchise values as intangible capital in order to avoid termination. The introduction of this notional capital is an example of ex-ante regulatory forbearance.

7 Conclusion

This paper proposes a new approach to designing prudential regulation of banks as a mechanism to implement the socially optimal allocation. Under hidden choice of risk, private information on returns and limited commitment by the banker, and costly liquidation, we first derive the optimal incentive-feasible allocation which specifies consumption by the banker and the probability of liquidation by the regulator. We then show that the optimal dynamic allocation is implementable through the combination of a risk-based deposit insurance pre-
mium and a book-value capital regulation. The implementation results suggest that it is optimal to base bank capital regulation on book-value capital as in PCA, and that stochastic liquidation is preferable to deterministic liquidation in this model.

This paper extends the model of DeMarzo and Fishman (2002) by considering explicitly hidden choice of risk in addition to private information on returns and limited commitment by the banker. It shows that, when the cost of exerting a high level of effort is larger than the corresponding improved expected return, it can be socially optimal to induce a low level of effort when a bank is well capitalized, but a high level of effort when it is undercapitalized. This result suggests that the regulator induce a high or low level of effort by the banker conditional on the level of bank capital.

Finally, it should be noted that the capital-ratio thresholds in the current Basel-type regulation are based on a value-at-risk method. Therefore, it is not surprising that many papers on bank capital regulation assume exogenously given capital-ratio thresholds. This paper, by contrast, derives the capital-ratio thresholds endogenously from the continuation function in a dynamic model. The regulator determines the threshold levels of capital based on whether it is beneficial for the regulator to liquidate a bank or not, and whether a bank’s performance is good enough to pay dividends.

This paper can be extended in a number of ways. First, it would be interesting to incorporate the possibility of recapitalization by the banker, which would change the level of book-value capital. In practice, it is very costly to issue new equity when a bank is classified as being undercapitalized. Further, the bank equity holders may be reluctant to issue new equity, since the benefit of recapitalization may end up mostly with the regulator and the debt holders of the bank. Second, it would be interesting to consider the optimal regulation in a multi-bank-one-regulator setup where there are positive and negative spillover effects of the closure of one bank on the other banks. Finally, in this paper, we assume that a bank cannot borrow from the market. An interesting extension would be the introduction of possible wholesale market funding for the bank.33
Appendix: Proofs of Propositions 1 and 2 and Corollary

Proof of Proposition 1.

(Step 1)

Let $v_c^t(\cdot)$ be the continuation function just before the banker and the FDIC consume and either the banker or the FDIC terminates the bank. Suppose a concave continuation function $v_t(\cdot)$ at the end of the period $t$ is given. Then, the optimal consumption is derived from the following problem:

$$v_c^t(w_c^t) = \max_{c_t(\cdot)} v_t(w_t) - c_t$$

such that $w_t + c_t = w_c^t$

$$c_t \geq 0.$$  

The FDIC will use the cheaper method between providing the banker with one unit of consumption and promising him one unit of continuation utility. Since $v_t(\cdot)$ is concave, we get $c_t(w_c^t) = \max(w_c^t - \overline{w}_t, 0)$ and $w_t = \min(w_t, \overline{w}_t)$.

Since the banker has the outside option $J_t = 0$, if $w_c^t < 0$, he will terminate the bank. So $v_c^t$ will be defined for $w_c^t \geq 0$. Since $v_t(w_t)$ is defined for $w_t \geq \underline{w}_t$, $v_t(w_t) = -\infty$ for $0 < w_t < \underline{w}_t$, and $(0, L)$ is located to the left of $v_t(\cdot)$, the FDIC can expand the continuation function using stochastic termination. The FDIC can enjoy all values of its continuation utility on the convex hull of $v_t(\cdot)$ and $(0, L)$. Given $w_c^t \leq \underline{w}_t$, in an optimal allocation, the FDIC either terminates the bank and sets $w_t = 0$ with probability $p_t(w_c^t) = (\underline{w}_t - w_c^t)/\underline{w}_t$, or allows the bank to continue operation and sets $w_t = \underline{w}_t$ with probability $1 - p_t(w_c^t)$. Therefore, after the banker consumes $c_t$ and survives stochastic termination, the relevant range of $w_t$ becomes an interval $[\overline{w}_t, \underline{w}_t]$. Finally, $v_c^t(\cdot)$ is also concave.

(Step 2)

< Case 1: $\mu_1 - \mu_0 \geq \psi >
Given $v^c_t(\cdot)$, we consider the continuation function $v^y_t(\cdot)$ before the choice of $e_t$ and the realization of $Y_t$. The FDIC cannot observe the choice of $e_t$ and the realization of $Y_t$, but wants the banker to choose $e_t = 1$ and $\hat{y}_t = y_t$. Thus, the FDIC must provide the banker with an incentive to exert high effort and report truthfully, by appropriately choosing the continuation utility function $w^c_t(\cdot)$. Finally, $w^e_t(Y_t; w^y_t)$ should be chosen to maximize the FDIC’s expected utility. Thus, we need to solve the following main problem:

$$
v^y_t(w^y_t) = \max_{w^c_t(Y_t)} E^1[Y_t + v^c_t(w^c_t(Y_t))] \\
\text{s.t. } E^1[w^c_t(Y_t)] - \psi \geq E^0[w^c_t(Y_t)] \quad (IC1) \\
w^c_t(Y_t) \geq w^c_t(y) + Y_t - y, \ \forall y \leq Y_t \quad (IC2) \\
E^1[w^c_t(Y_t)] - \psi = w^y_t. \quad (PK)
$$

The first constraint is an incentive-compatibility constraint for the banker such that it is optimal to exert high effort. The second is the other incentive-compatibility constraint such that it is optimal for the banker to tell the truth about the realization of returns. The last constraint is the promise-keeping constraint for the banker.

Before we solve the above problem, we define the following relaxed problem:

$$
v^y_t(w^y_t) = \max_{w^c_t(Y_t)} E^1[Y_t + v^c_t(w^c_t(Y_t))] \\
\text{s.t. } w^c_t(Y_t) \geq w^c_t(y) + Y_t - y, \ \forall y \leq Y_t \quad (IC2) \\
E^1[w^c_t(Y_t)] - \psi = w^y_t. \quad (PK)
$$

To solve the relaxed problem, note that from (IC2), $w^c_t(y) \geq 1$ should hold. Given $w^y_t$, the promise-keeping constraint determines the mean of the random function $w^c_t(Y_t)$. Since $v^c_t(\cdot)$ is concave, it is optimal to minimize the variability of $w^c_t(Y_t)$. The solution of the relaxed problem is thus $w^c_t(Y_t) = w^y_t + \psi - \mu_1 + Y_t$. When we substitute $w^c_t(Y_t) = w^y_t + \psi - \mu_1 + Y_t$ into (IC1), (IC1) holds. Thus, the solution of the main problem is $w^c_t(Y_t) = w^y_t + \psi - \mu_1 + Y_t$. Once we have $v^y_t(w^y_t) = \mu_1 + E^1[v^c_t(w^c_t(Y_t))]$, $v^y_t(\cdot)$ is also concave.

< Case 2: $\mu_1 - \mu_0 < \psi >$
Now in every period the FDIC chooses to induce either $e_t = 1$ or $e_t = 0$ depending on the value of $w_t^g$. summarizing the history. The FDIC’s choice of $w_t^c(Y_t)$ is more complicated since we need to consider the possibility of a randomization by the FDIC to concavify the continuation function. Given $v_t^c(\cdot)$, we consider the continuation function $v_t^g(\cdot)$ before the choice of $e_t$ and the realization of $Y_t$.

First, we calculate the continuation function when the FDIC wants the banker to choose $e_t = 1$ and $\widehat{y}_t = y_t$. We need to solve the following main problem:

$$v_t^g(w_t^g; e_t = 1) = \max_{w_t^c(Y_t)} E^1[Y_t + v_t^c(w_t^c(Y_t))]$$

s.t. \[E^1[w_t^c(Y_t)] - \psi \geq E^0[w_t^c(Y_t)]\] (IC1)

\[w_t^c(Y_t) \geq w_t^c(y) + Y_t - y, \ \forall y \leq Y_t\] (IC2)

\[E^1[w_t^c(Y_t)] - \psi = w_t^g.\] (PK)

This problem is the same as in Case 1, but now the solution of the relaxed problem violates (IC1). Thus, the FDIC needs to choose a different solution. From (IC2), we get $w_t^c(Y_t) = \alpha + \gamma Y_t$, where $\gamma \geq 1$. From the promise-keeping constraint, $E^1[\alpha + \gamma Y_t] - \psi = w_t^g$. Thus, $w_t^c(Y_t) = w_t^g + \psi + \gamma(Y_t - \mu_1)$. Substituting this into (IC1), we get $\gamma(\mu_1 - \mu_0) \geq \psi$. Since $v_t^c(\cdot)$ is concave and higher $\gamma$ implies a mean-preserving spread in $w_t^c$, the FDIC will pick the lowest possible $\gamma$ satisfying (IC1) and (IC2) in order to minimize the variability of $w_t^c$. Thus, the solution will have $\gamma = \psi/(\mu_1 - \mu_0)$, and $w_t^c(Y_t) = w_t^g + \psi + (\psi/ (\mu_1 - \mu_0)) (Y_t - \mu_1)$. Once we have $v_t^g(w_t^g; e_t = 1) = \mu_1 + E^1[v_t^c(w_t^c(Y_t))], v_t^g(\cdot; e_t = 1)$ is also concave.

Second, we calculate the continuation function when the FDIC wants the banker to choose $e_t = 0$ and $\widehat{y}_t = y_t$. We need to solve the following problem:

$$v_t^g(w_t^g; e_t = 0) = \max_{w_t^c(Y_t)} E^0[Y_t + v_t^c(w_t^c(Y_t))]$$

s.t. \[E^0[w_t^c(Y_t)] \geq E^1[w_t^c(Y_t)] - \psi\] (IC1)

\[w_t^c(Y_t) \geq w_t^c(y) + Y_t - y, \ \forall y \leq Y_t\] (IC2)

\[E^0[w_t^c(Y_t)] = w_t^g.\] (PK)
The first constraint is an incentive-compatibility constraint for the banker such that it is optimal to exert low effort. The second is the truth-telling constraint. The last constraint is the promise-keeping constraint for the banker.

Before we solve the above problem, we define the following relaxed problem:

\[ v_t^y(w_t^y; e_t = 0) = \max_{w_t^y(Y_t)} E^0[Y_t + v_t^y(w_t^y(Y_t))] \]

subject to

\[ w_t^y(Y_t) = w_t^y - \mu_0 + Y_t, \quad \forall y \leq Y_t \quad (IC2) \]

\[ E^0[w_t^y(Y_t)] = w_t^y. \quad (PK) \]

The solution of the relaxed problem is \( w_t^y(Y_t) = w_t^y - \mu_0 + Y_t \). When we substitute \( w_t^y(Y_t) = w_t^y - \mu_0 + Y_t \) into \((IC1)\), \((IC1)\) holds. Thus, the solution of the main problem is \( w_t^y(Y_t) = w_t^y - \mu_0 + Y_t \). Once we have \( v_t^y(w_t^y; e_t = 0) = \mu_0 + E^0 [v_t^y(w_t^y(Y_t))] \), \( v_t^y(\cdot; e_t = 0) \) is also concave.

Therefore, we have

\[ v_t^y(w_t^y; e_t = 1) = E^1[Y_t + v_t^y(w_t^y + \psi + (\psi/ (\mu_1 - \mu_0)) (Y_t - \mu_1))] \]

\[ v_t^y(w_t^y; e_t = 0) = E^0[Y_t + v_t^y(w_t^y - \mu_0 + Y_t)]. \]

In the second stage in period \( t \) given \( v_t^y(\cdot) \), if the FDIC wants the banker to choose \( e_t = 0 \) and \( \hat{y}_t = y_t \), it will prescribe \( w_t^y(Y_t) = w_t^y - \mu_0 + Y_t \), while if it wants him to choose \( e_t = 1 \) and \( \hat{y}_t = y_t \), it will prescribe \( w_t^y(Y_t) = w_t^y + \psi + (\psi/ (\mu_1 - \mu_0)) (Y_t - \mu_1) \). Thus, given a value of \( w_t^y \), the FDIC will induce \( e_t = 0 \) if \( E^1[Y_t + v_t^y(w_t^y + \psi + (\psi/ (\mu_1 - \mu_0)) (Y_t - \mu_1))] \leq E^0[Y_t + v_t^y(w_t^y - \mu_0 + Y_t)] \), and \( e_t = 1 \) if \( E^1[Y_t + v_t^y(w_t^y + \psi + (\psi/ (\mu_1 - \mu_0)) (Y_t - \mu_1))] \geq E^0[Y_t + v_t^y(w_t^y - \mu_0 + Y_t)]. \]

Let \( \tilde{v}_t^y(w_t^y) = \max[v_t^y(w_t^y; e_t = 1), v_t^y(w_t^y; e_t = 0)] \). Moreover, since \( v_t^y(\cdot; e_t = 1) \) and \( v_t^y(\cdot; e_t = 0) \) are both concave, \( \tilde{v}_t^y(\cdot) \) is not concave whenever \( v_t^y(\cdot; e_t = 1) \) and \( v_t^y(\cdot; e_t = 0) \) intersect. Thus, the FDIC is willing to introduce a randomization scheme to concavify the continuation function and enjoy higher utility. We denote by \( v_t^y(\cdot) \) the convex hull of \( \tilde{v}_t^y(\cdot) \). In particular, if \( v_t^y(w_t^y; e_t = 1) > v_t^y(w_t^y; e_t = 0) \), \( \forall w_t^y \), then \( \tilde{v}_t^y(w_t^y) = v_t^y(w_t^y; e_t = 1), \forall w_t^y \), and if \( v_t^y(w_t^y; e_t = 1) < v_t^y(w_t^y; e_t = 0) \), \( \forall w_t^y \), then \( \tilde{v}_t^y(w_t^y) = v_t^y(w_t^y; e_t = 0), \forall w_t^y \). More interestingly, suppose \( v_t^y(\cdot; e_t = 1) \) and \( v_t^y(\cdot; e_t = 0) \) intersect at \( w_t^y = \tilde{w}_t^y \). Let \( \tilde{w}_t^{y1} \) be the value of \( w_t^y \) of the tangent point on the convex hull associated with \( v_t^y(\cdot; e_t = 1) \), and \( \tilde{w}_t^{y0} \) be the value of \( w_t^y \) of
of the tangent point on the convex hull associated with \( v^y_t(\cdot; e_t = 0) \). Then, when \( w^y_t \leq \overline{w}^y_t \), \( v^y_t(w^y_t) = v^y_t(w^y_t; e_t = 1) \), and when \( w^y_t \geq \overline{w}^y_t \), \( v^y_t(w^y_t) = v^y_t(w^y_t; e_t = 0) \). Also, we denote by \( \hat{p}_t \) the probability used to construct the convex hull. In particular, if \( \overline{w}^y_t < w^y_t < \underline{w}^y_t \), \( \hat{p}_t = (\overline{w}^y_t - w^y_t)/(\overline{w}^y_t - \underline{w}^y_t) \). Then, for \( \overline{w}^y_t < w^y_t < \underline{w}^y_t \), \( v^y_t(w^y_t) = \hat{p}_t v^y_t(\overline{w}^y_t) + (1 - \hat{p}_t) v^y_t(\underline{w}^y_t) \).

Figure A.1 shows an example in which the FDIC chooses a randomization over the two continuation functions. Finally, \( v^y_t(w^y_t) \) is also concave since it is a convex hull of concave functions.

(Step 3)

Moving from period \( t \) to period \( t - 1 \) involves discounting the continuation utilities of the banker and the FDIC, so \( w_{t-1} = \beta w^y_t \) and \( v_{t-1} = \delta v^y_t \). Given \( v^y_t(\cdot) \), the continuation function at the end of period \( t - 1 \) is \( v_{t-1}(w_{t-1}) = \delta v^y_t(\beta^{-1} w_{t-1}) \). Finally, \( v_t(\cdot) \) is also concave. Note that \( \overline{w}^y_{t-1} \equiv \beta \overline{w}^y_t \), and \( \underline{w}^y_{t-1} \equiv \beta \underline{w}^y_t \).

So far, we have solved for the continuation functions starting from a concave \( v_t(\cdot) \) to a concave \( v_{t-1}(\cdot) \). Suppose that we have a finite horizon with \( T < \infty \). Then, \( v_T(w_T) \) is linear. Thus, starting from the weakly concave \( v_T(w_T) \), we can recursively solve for the continuation function at all stages in all periods down to \( v_0^y(w_0^y) \), which is also concave. When \( T \to \infty \), by the Contraction Mapping Theorem, we get the time-invariant continuation function \( v(\cdot) \) as a unique fixed point, and thus \( v(\cdot) \) is also concave.

The long-run behavior of the optimal allocation is determined by the dynamics of the state variable \( w \) over time and stages. The evolution of the state variable is as follows:

(1) If \( \mu_1 - \mu_0 \geq \psi \),
\[
  w_{t-1} \Rightarrow w^c_t = \beta^{-1} w_{t-1} + \psi - \mu_1 + y_t \Rightarrow w_t = \min(\overline{w}_t, \max(w^c_t, w^y_t)) \quad \text{if } \lambda_{t+1} = 0
\]
\[
  \text{or } 0 \quad \text{if } \lambda_{t+1} = 1;
\]

(2) If \( \mu_1 - \mu_0 < \psi \),

(i) when \( w_{t-1} \leq \overline{w}^c_{t-1} \),
\[ w_{t-1} \Rightarrow w_t = \beta^{-1} w_{t-1} + \psi + \frac{\psi}{\mu_1-\mu_0} (y_t - \mu_1) \Rightarrow w_t = \min(w_t, \max(w_t, w_t)) \]

if \( \lambda_{t+1} = 0 \)

or \( 0 \)

if \( \lambda_{t+1} = 1 \);

(ii) when \( w_{t-1} \geq \bar{w}_{t-1} \),

\[ w_{t-1} \Rightarrow w_t = \beta^{-1} w_{t-1} + y_t - \mu_0 \Rightarrow w_t = \min(w_t, \max(w_t, w_t)) \]

if \( \lambda_{t+1} = 0 \)

or \( 0 \)

if \( \lambda_{t+1} = 1 \);

(iii) when \( \bar{w}_{t-1} < w_{t-1} < \bar{w}_{t-1} \),

\[ w_{t-1} \Rightarrow w_t = \rightbraceoverset{\text{with prob. } \tilde{p}_t}{\text{if } \lambda_{t+1} = 0} \]

or \( \rightbraceoverset{\text{or } \bar{w}_{t-1} + y_t - \mu_0}{\text{with prob. } 1 - \tilde{p}_t} \)

if \( \lambda_{t+1} = 1 \).

**Proof of Corollary 1.**

In the first stage of period 0, the FDIC chooses the initial transfer \( K_0 \) and the initial continuation utility of the banker \( w^c_0 \) to maximize its utility \( K_0 + v^c_0(w^c_0) \) subject to the banker’s participation constraint \( w^c_0 \geq K_0 \). Since the FDIC will absorb all the rent, it will choose \( K_0 = w^c_0 \) and the potential banker will accept the allocation. The optimal choice of \( K_0 \) by the FDIC is \( K^*_0 = w^c_0 \in [\bar{w}_0, \infty) \). However, if \( w^c_0 > \bar{w}_0 \), in the second stage of period 0, the FDIC will choose \( c_0(w^c_0) = w^c_0 - \bar{w}_0 \) and \( w^c_0 = \bar{w}_0 \) holds. Thus, the FDIC, without loss of generality, chooses \( K^*_0 = w^c_0 = w^*_0 = \bar{w}_0 \).

**Proof of Proposition 2.**

(Part 1)

In order to replicate the behavior of the optimal allocation, we need to find a deposit insurance premium \( \tilde{x}_t \) such that the law of motion of the level of book-value capital \( K_t \) replicates the law of motion of the banker’s continuation utility \( w_t \). In the initial period, the banker sets aside the initial capital \( K_0 = K_0 = \bar{w}_0 \) in the form of cash. Then, the FDIC
allows the banker to take deposit of one dollar and invest it into the risky assets. In period 1, from the assumption that the banker uses all realized return \( y_1 \) to increase the level of capital and the assumption that capital is held with cash, after the banker pays the deposit insurance premium, the level of capital becomes \( K^d_1 = K_0 + y_1 - \tilde{x}_1 \). Let \( \tilde{x}_1 = \mu_1 - \psi - K_0(\beta^{-1} - 1) \), when \( \mu_1 - \mu_0 \geq \psi \). Then \( K^d_1 = w^c_1 \). Now when \( \mu_1 - \mu_0 < \psi \), we need to consider three different subcases. First, when \( K_0 < \overline{K}_1 \), we set \( \tilde{x}'_1 = (\psi / (\mu_1 - \mu_0)) \mu_0 - K_0(\beta^{-1} - 1) - y_1(\psi / (\mu_1 - \mu_0) - 1) \). Second, when \( K_0 > K_0^0 \), we set \( \tilde{x}'_1 = \mu_0 - K_0(\beta^{-1} - 1) \). Third, when \( \overline{K}_1 \leq K_0 \leq K_0^0 \), we set \( \tilde{x}'_1 = [(\psi / (\mu_1 - \mu_0)) \mu_0 - K_0(\beta^{-1} - 1) - y_1(\psi / (\mu_1 - \mu_0) - 1)] + [\beta^{-1}(K_0 - \overline{K}_1)] \) with probability \( \hat{p}_1 \), and \( \tilde{x}'_1 = [\mu_0 - K_0(\beta^{-1} - 1)] - [\beta^{-1}(K_0^0 - K_0)] \) with probability \( 1 - \hat{p}_1 \), where \( \hat{p}_1 = (K_0^0 - K_0) / (K_0^0 - \overline{K}_1) \). Then we get \( K^d_1 = w^c_1 \).

Now the termination probability \( p_1 = \max([K_1 - K^d_1] / K_1, 0) \) is the same as the termination probability \( p^*_1 \) in the ex-ante optimal allocation, the dividend \( d_1 = \max(K^d_1 - K_1, 0) \) is the same as the banker’s consumption \( c^*_1 \), and the randomization over two levels of efforts \( \hat{p}_t \) is also the same as the optimal randomization \( \widehat{p}^*_t \) in the ex-ante allocation. In the optimal allocation, if the bank survives stochastic termination, the FDIC provides a higher continuation utility \( w_1 \). Likewise, in this implementation, the FDIC provides a subsidy or recapitalization, so that the level of capital becomes \( K_1 = w_1 \). After dividend payments and stochastic termination, \( K_1 = w_1 \) holds. This can be repeated in all subsequent periods \( t \geq 2 \). Therefore, the laws of motion of \( K^d_t \) and \( K_t \) replicate those of \( w^c_t \) and \( w_t \), respectively, and we get \( d_1(K_0, y_1) = c^*_1(w_0, y_1) \) and \( p_1(K_0, y_1) = p^*_1(w_0, y_1) \).

(Part 2)

< Case 1: \( \mu_1 - \mu_0 \geq \psi \)>

Along the equilibrium path with \( e_t = 1 \) and \( y^t = y^t \), the banker’s continuation utility at the end of period \( t \) is given by \( w_t = K_t \). Since \( K_t \geq K^d_t = w_t \), we have \( w_t \geq w_t > J_t = 0 \), so that the banker has no incentive to quit early or become undercapitalized without fully using capital. From the definition of \( K^d_t \), every dollar that the banker uses for private consumption, rather than for building up capital, leads to a reduction of \( w^c_t \) and \( K^d_t \) by one dollar. This
is true even if the banker eats capital up to the point where $K_t^d$ is below $K_t$. Therefore, the banker has no incentive to underaccumulate capital, and the strategy of the banker to use all realized profits to accumulate capital every period is optimal. Finally, given that $\tilde{y}_t = y_t$, the banker will choose $e_t = 1$, since $E_t^1 [d_t - \psi + w_t] > E_t^0 [d_t + w_t]$ holds for all $t$.

$< \text{Case 2: } \mu_1 - \mu_0 < \psi, K_{t-1} < K^1_t >$

As in the proof for Case 1, the banker has no incentive to quit early or become undercapitalized without fully using capital. From the definition of $K^d_t$, every dollar that the banker uses for private consumption, rather than for building up capital, leads to a reduction of $w^c_t$ and $K^d_t$ by $(\psi / (\mu_1 - \mu_0)) (> 1)$ dollar. Therefore, the banker will use all realized profits to accumulate capital. Finally, given that $\tilde{y}_t = y_t$, it is weakly optimal for the banker to choose $e_t = 1$, since $E_t^1 [d_t - \psi + w_t] = E_t^0 [d_t + w_t]$ holds.

$< \text{Case 3: } \mu_1 - \mu_0 < \psi, K^0_t < K_{t-1} >$

Similarly, the banker has no incentive to quit early or become undercapitalized without fully using capital. From the definition of $K^d_t$, every dollar that the banker uses for private consumption, rather than for building up capital, leads to a reduction of $w^c_t$ and $K^d_t$ by one dollar. Therefore, the banker will use all realized profits to accumulate capital. Finally, given that $\tilde{y}_t = y_t$, the banker will choose $e_t = 0$, since $E_t^0 [d_t + w_t] > E_t^1 [d_t - \psi + w_t]$ holds.

$< \text{Case 4: } \mu_1 - \mu_0 < \psi, K^1_t < K_{t-1} < K^0_t >$

Similarly, the banker has no incentive to quit early or become undercapitalized without fully using capital. From the definition of $K^d_t$, every dollar that the banker uses for private consumption, rather than for building up capital, leads to a reduction of $w^c_t$ and $K^d_t$ by either one dollar or $\psi / (\mu_1 - \mu_0)$ dollar. Therefore, the banker will use all realized profits to accumulate capital. Finally, given that $\tilde{y}_t = y_t$, when the FDIC decides to induce $e_t = 1$ following randomization and charges $\left[ \beta^{-1} (K_{t-1} - K^1_t) \right]$ to the banker, it is weakly optimal for the banker to choose $e_t = 1$, since $E_t^1 [d_t - \psi + w_t] = E_t^0 [d_t + w_t]$ holds. On the other hand, when the FDIC decides to induce $e_t = 0$ following randomization and subsidizes $\left[ \beta^{-1} (K^0_t - K_{t-1}) \right]$ to the banker, the banker will choose $e_t = 0$, since $E_t^0 [d_t + w_t] > E_t^1 [d_t - \psi + w_t]$ holds. ■
Notes

1In this paper, the regulator acts in the interest of taxpayers, so that there is no principal-agent problem between the regulator and taxpayers as mandated in the FDICIA. Moreover, as modeled by Akerlof and Romer (1993), the FDIC fully guarantees the deposits of a bank. This is equivalent to the depositors holding government debt and the government lending money directly to the bank. Dewatripont and Tirole (1994) propose that the goal of banking regulation be to actively represent small depositors who are unwilling or unable to monitor banks. This paper emphasizes that small depositors protected by deposit insurance are also taxpayers, who are responsible for the losses in a deposit insurance fund. In this sense, the FDIC is the representative of small depositors and taxpayers.

2In practice, the FDIC is not likely to have full information on return distributions.

3DeMarzo and Fishman (2002) only consider the case where high effort is socially optimal in the first-best sense. In this paper, we also consider the other case where low effort is optimal in the first-best sense. This case has not been fully examined in the recent literature. For example, DeMarzo and Sannikov (2006) assume that high effort is efficient. Biais et al (2007) also extend DeMarzo and Fishman (2002), but do not consider the hidden choice of effort.

4This is a strong assumption, considering that most US banks are subject to regulatory supervision and regular auditing. Smith and Wang (1998) have a model of two-period repeated insurance relationships in which the insurer can observe the bank’s income realization by incurring a fixed monitoring cost.

5If we relax some of the above assumptions, then the incentive-feasible allocation might change. For example, if we assume that the FDIC has only partial information on the return distributions, there is room for adverse selection. Then the incentive-compatible allocation should satisfy an additional truth-telling constraint, which would make it more difficult for the FDIC to provide the right incentives to the banker. Also, if the FDIC has access to a costly state verification technology instead of relying solely on the report by the banker, I expect the optimal allocation will be similar to that shown by Monnet and Quintin (2005): for any history, either the agent transfers the project’s entire revenues to the principal, or the principal promises to require no further transfers in the future.

6Heaton et al (2010) provide an explanation of the conceptual differences between mark-to-market, or fair value, accounting and historical cost, or book value, accounting.

7It should be noted that the banker’s continuation utility is the expected value of future dividends discounted by a constant discount factor. This contrasts with book-value capital, which is the sum of past savings by the banker. Because this paper does not model what is observable to the market and there is no
mechanism to transfer the value directly to outside investors, market-value capital of the bank is not well defined. However, under the assumptions that outside investors are risk-neutral and that outsider investors and the banker share the same constant discount factor, the banker’s continuation utility, which is the present value of non-traded future dividends, can be viewed as a proxy for market-value capital.

Isaac (2000) points out that the FDIC in practice collects premiums from banks and thrifts, and turns them over to the Treasury.

The main results do not change when we assume a finite horizon.

\( Y_t \) can be interpreted as net income after obligations to depositors are met. The assumption of intertemporally independent income is made for ease of analysis, and does not necessarily fit the time-series properties of bank return data. Also, note that even though a long-term project itself may generate intertemporally independent returns, the actual long-term bank-borrower contract can introduce persistence into bank return.

I thank both referees for their suggestions regarding these points.

We can generalize the choice of effort by letting the banker choose \( e \in [0, \bar{e}] \), but need to assume \( \partial F(x \mid e_t)/\partial e_t > 0, \forall x, \forall e_t \).

We assume that the banker has limited liability, so that \( c_t \geq 0 \), but the FDIC does not. Note that allowing for no negative consumption is equivalent to setting the utility of a negative consumption to \(-\infty\), as in Cole and Kocherlakota (2001).

Proposition 1 in Section 3.2 shows that, under certain circumstances, we also need to consider a randomization \( \tilde{p}_t \) over effort levels.

Since the banker can terminate the bank anytime in period \( t \) and since \( c_t \geq 0 \), he can wait until the FDIC decides to terminate the bank or not. Thus, we can without loss of generality assume that the banker terminates the bank after the FDIC’s termination decision.

Note that depending on the parameter values and history, the FDIC wants to induce either high effort or low effort. Proposition 1 shows exactly how the desirable level of effort \( e^*_t \) is determined.

We restrict attention to direct mechanisms. By the Revelation Principle, this is without loss of generality. Later in the implementation part of the paper, we show that \((e^*, \tilde{y}^*, q^*)\) is an equilibrium strategy for the banker given a specific mechanism.

In a similar setting, Piskorski and Tchistyi (2010) allow for a stochastic interest rate which the lender uses to discount future cash flows. Thus, when the interest rate jumps, the borrower’s continuation utility also jumps.
As will be shown later, when $w$ is large enough and $v$ is small enough, the optimal allocation is characterized by a positive dividend paid to the banker and the banker’s continuation utility reduced by the same amount. Therefore, $w_{lab}$ takes a finite value.

The proof that the principle of optimality holds in this problem is available to readers upon request.

We can show under what circumstances the FDIC wants to induce high effort, although low effort is first-best optimal. An example of Proposition 1 (3) is available to readers upon request.

We can instead assume that the FDIC is fully “captured” by the potential banker. Then, the potential banker will not pay any initial transfer and will choose $w^c_0$ at a high level, so the FDIC just breaks even. In this paper, we assume that the FDIC tries to minimize the tax burden in the initial period.

Note that the total amount of funds invested in the long-term assets is fixed as the amount of deposits $D$ during the life of a bank, so that $Y_t$ does not depend on the level of capital $K_t$.

When the FDIC terminates the bank with capital $K_t = w^c_t$, this amount belongs to the FDIC. By contrast, when the FDIC bails out the bank, it should pay $K_t - w^c_t$ to the bank in order to replenish capital. Therefore, in expectation, $(w_t - w^c_t) / w_t \cdot w_t^c - (w_t^c / w_t) \cdot (w_t - w^c_t) = 0$ and the FDIC breaks even.

Note that this implementation relates to both liquidity and capital management in the sense that book-value capital $K_t$ is liquid, while the risky project is illiquid. Considering that central banks in general make liquid assets available on demand to banks, we can consider the possibility of an alternative implementation in which a central bank provides a credit line to banks. Typically, a central bank lends to a solvent bank facing liquidity problems such as sudden deposit withdrawals or temporary funding difficulties, and lend against good collateral so that they don’t incur credit losses ex post. However, a proactive central bank might be willing to provide a credit line involving potential debt forgiveness to a bank subject to bad income shocks. The optimal allocation could be implemented by a flow of payments from the bank to the central bank, a credit line with an interest rate and a credit limit, and a stochastic termination rule under which the central bank stochastically terminates the bank or forgives part of its debt when the outstanding balance exceeds the credit limit. I thank a referee for suggestions.

The proof of the equivalence of stochastic and partial termination is available to readers upon request.

DeMarzo and Fishman (2003) use this assumption in modeling investment to alter the project scale.

I choose thrift institutions rather than commercial banks because thrift institutions are more likely to be terminated under PCA while large and systemically important commercial banks are less likely.
This result is derived from the assumptions of linear preferences and limited commitment. Note that in practice, it would be difficult to precisely estimate the support of a bank's return distribution.

I thank Deborah Lucas and a referee for their suggestions.

Note that partial termination does not involve bailouts by the FDIC.

Holland (1998, p.49) points out that the FDIC has the authority to arrange for the acquisition of all or a portion of a failed or failing bank, which is called deposit assumption, and that the term *purchase and assumption* is applied to the most prevalent type of deposit assumption.

I thank Deborah Lucas for pointing out this important issue and providing me with suggestions.

I thank a referee for suggesting this point.
(Table 1) FDIC Initial Base Assessment Rates (unit: basis point)

<table>
<thead>
<tr>
<th>CAMELS Rating 1, 2</th>
<th>CAMELS Rating 3</th>
<th>CAMELS Rating 4, 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well Capitalized</td>
<td>I (12~16)</td>
<td>III (32)</td>
</tr>
<tr>
<td>Adequately Capitalized</td>
<td>II (22)</td>
<td></td>
</tr>
<tr>
<td>Undercapitalized</td>
<td>III (32)</td>
<td>IV (45)</td>
</tr>
</tbody>
</table>

Source: FDIC website.
(Table 2) Classification of Banks in PCA

<table>
<thead>
<tr>
<th></th>
<th>Total Risk-Based Capital Ratio</th>
<th>Tier 1 Risk-Based Capital Ratio</th>
<th>Leverage Ratio</th>
<th>Tangible Equity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well Capitalized</td>
<td>≥10% and ≥ 6% and ≥ 5% and &gt; 2%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adequately Capitalized</td>
<td>≥ 8% and ≥ 4% and ≥ 4% and &gt; 2%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Undercapitalized</td>
<td>≥ 6% or ≥ 3% or ≥ 3% and &gt; 2%</td>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Significantly Undercapitalized</td>
<td>&lt; 6% or &lt; 3% or &lt; 3% and &gt; 2%</td>
<td>or</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critically Undercapitalized</td>
<td>- or - or - and ≤ 2%</td>
<td>or</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Tier 1 or core capital mainly comprises permanent shareholders’ equity. Tier 2 or supplementary capital comprises loan loss reserves, subordinated debts, asset revaluation reserves, hybrid capital instruments, etc. Total capital is the sum of Tier 1 and Tier 2 capital. See Basel Committee on Banking Supervision (1988) for the formal definitions of capital. The total risk-based capital ratio is the ratio of total capital to risk-weighted assets. Risk-weighted assets are calculated by applying different risk weights to each type of assets. The Tier 1 risk-based capital ratio is the ratio of Tier 1 capital to risk-weighted assets. The leverage ratio is the ratio of Tier 1 capital to total assets. The tangible equity ratio is the ratio of ‘the total of Tier 1 capital plus cumulative preferred stock and related surplus less intangibles except qualifying purchased mortgage servicing rights’ to ‘the sum of total assets less intangible assets except qualifying purchased mortgage servicing rights’.

Sources: Federal Deposit Insurance Act, FDIC rules and regulations.
Figure 1: Determination of the Optimal Allocation when 

\[ \mu_1 - \mu_0 > \psi. \]
Figure 2: Determination of the Optimal Allocation when

\[ \mu_1 - \mu_0 < \psi. \]
Figure 3: Timing of Events
Figure 4: Liquidation Value and Continuation Function
Figure 5: Liquidation Value and Capital Thresholds
Figure 6: Mean-Preserving Spread and Continuation Function
Figure A.1: Continuation Function with Randomization over Effort
Literature Cited


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