Estimating Probabilities of Recession in Real Time
Using GDP and GDI

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Abstract

This work estimates Markov switching models on real time data and shows that the growth rate of gross domestic income (GDI), deflated by the GDP deflator, has done a better job recognizing the start of recessions than has the growth rate of real GDP. This result suggests that placing an increased focus on GDI may be useful in assessing the current state of the economy. In addition, the paper shows that the definition of a low-growth phase in the Markov switching models changed considerably from 1978 to 2005. The models increasingly came to define this phase as an extended period of around zero rather than negative growth, diverging somewhat from the traditional definition of a recession.

JEL classification: C22, E32, E37.

Keywords: Business cycles, recession probabilities, Markov switching models, real-time data analysis

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1 Introduction

Since Hamilton (1989, 1990), many authors have computed probabilities of recession using Markov switching models for real gross domestic product (GDP). The popularity of these models stems in part from the fact that their estimated probabilities of recession line up well with the peak and trough dates produced by the NBER’s business cycle dating committee. However when Chauvet and Hamilton (2005) use databases on real GDP from the Federal Reserve Bank of Philadelphia to examine the performance of Markov switching models in real time, their performance is more mixed, and was quite poor in recognizing the start of the 2001 recession.

Overlooked so far in this literature is the fact that, while the U.S. Bureau of Economic Analysis (BEA) press releases feature GDP, its expenditure-based estimate of the size of the economy, BEA also produces an income-based estimate of the size of the economy, gross domestic income (GDI).\(^1\) GDI should equal GDP, theoretically, but in practice they often diverge substantially. The appeal of exploiting the information in GDI to date recessions is simple: it is as comprehensive as GDP, but it may capture information about the economy missed by measured GDP. For example, GDI may capture informative variation in income and profits not fully reflected by GDP. Grimm (2005) has shown that GDI tends to fall more than GDP in recessions, a sign that GDI may be useful in making inferences about recessions.

This paper examines the performance of GDI for making inferences about recessions in real time. It estimates univariate Markov switching models for the growth rates of real GDI and real GDP, and bivariate Markov switching models that employ the information

\(^1\)The BEA may emphasize GDP because GDP is produced in a more timely manner than GDI, a fact whose implications are discussed later in the paper. The BEA may also emphasize GDP because they produce price deflators for the detailed components of GDP but not GDI.
in both. It then compares the real time estimated probabilities of recession from these models to various benchmarks, such as the NBER start and end dates for recessions and smoothed probabilities of recession from the Markov switching models themselves.\textsuperscript{2} The main finding is that, no matter what benchmark one uses, real time GDI has done a substantially better job recognizing the start of the last several recessions than has real time GDP. This result stems in part from the fact that GDI tends to fall more than GDP in recessions, but also because the relatively small variance of GDI in the Markov switching models, conditional on the estimated state of the world, makes it more reliable than GDP as a signal of the state. However, I also point out reasons why GDP remains important to consider in tandem with GDI in assessing the current state of the economy.

As a secondary finding, the paper shows that the parameters estimated from the Markov switching models have changed in interesting ways over the past couple of decades. At the start of the out-of-sample evaluation period, in 1978, the models defined their low-growth state as a period with substantially negative mean growth, and their smoothed probabilities of the low-growth state lined up well with the NBER’s recession dates. However, estimated mean growth in the low-growth state crept up considerably from 1978 to 2005 to around zero (see also Kim and Nelson, 1999b), with smoothed probabilities showing low-growth phases that are substantially longer than recessions as defined by the NBER. So it is important to note that while I continue to use the short-hand term “recession” to describe low-growth periods as identified by the models, by the end of the sample used in this paper, the models came to define these periods in a way that diverges from the traditional meaning of the term.

\textsuperscript{2}These smoothed probabilities, also called two-sided probabilities, use time series realizations of GDP and GDI from the future as well as the past to estimate the probability of recession at any given date, and hence are more accurate than the one-sided probabilities using only past realizations available in real time.
Section 2 shows estimated probabilities of recession for the most recent quarter from 1978 to 2005 for selected BEA releases. Section 3 examines time series of these smoothed probabilities around the recessions in the sample. Section 4 concludes.

2 Probabilities of Recession Using Real Time Data

Figure 1 shows real time estimated probabilities of recession computed using the univariate model in Chauvet and Hamilton (2005) employing real GDP growth $\Delta y^1_t$. There are two states of the world, expansion ($S_t = 1$) and recession ($S_t = 2$). The mean growth rate $\mu^1_{s_t}$ switches depending on the state of the economy; however each state is assumed to have a common variance $(\sigma^1)^2$. The figure shows the probability for the latest quarter available at each of the BEA’s “final” current quarterly data releases - their third estimate for each quarter, released about 3 months after the quarter ended - from 1978Q1 to 2005Q3. Prior to 1991Q4, we use real GNP growth instead of real GDP growth. Each time series extends back to 1959Q4. The probability for 1978Q1, then, is the probability of recession for that quarter computed using the real GNP growth time series from 1959Q4 to 1978Q1 that existed at the end of June 1978. Recessions as defined by the NBER shaded gray. This probability shoots up well above 50% at some point during the course of each NBER recession, but in the starting quarter the signal

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3While the two state model is most popular in the literature, a three state “plucking” model, with GDP bouncing back rapidly after a recession, has been examined quite a bit as well: see Friedman (1993), Sichel (1994), and Kim and Nelson (1999a). The 1990-91 and 2001 recessions, exhibiting periods of slow growth after the NBER-defined end date, argue against the use of the three state model in analyzing recent history.

4We shade areas in a somewhat expansive way; the NBER dates the beginning and end of recessions by month; if any month of a quarter is classified by the NBER as in recession, we shade the entire quarter. For example, the NBER says the last recession in the plots started in March 2001 and ended in November 2001; we shade both 2001Q1 and 2001Q4.
is typically unclear. At the NBER-defined start of the 1980, 1981-82, 1990-91, and 2001 recessions, the latest probabilities were 52%, 40%, 45%, and 23%, respectively. In the latest recession, the real time performance of the model is particularly poor, giving us 23% and 39% probabilities in its first two quarters.

Table 1 shows the first set of parameter estimates of the model, computed using the time series of real GNP growth available in 1978Q1, and the last set of estimates, computed using the time series of real GDP growth available in 2005Q3. Standard errors are in parentheses. The last set of estimates, computed using revised data and the realizations of several additional recessions, has a slightly lower mean growth rate during expansions and a substantially higher mean growth rate during recessions, approximately equal to zero. This contraction of the mean spread $\mu_{S_t=1}^{1} - \mu_{S_t=2}^{1}$, as well as the slight increase in the conditional variance $(\sigma^{1})^{2}$, imply that it has become more difficult to recognize recessions over the last 27 years using real GDP, as its behavior in the different states has become less distinct. All of the Great Moderation - the moderation in the volatility of economic activity from the mid-1980s to mid-2000s (e.g., McConnell and Perez-Quiros, 2000) - can be attributed to the reduction in the spread between the mean growth rates in high- and low-growth states. See Kim and Nelson (1999b) for more on this point.

Table 2 shows the first and last parameter estimates of the same univariate model substituting real GDI, $\Delta y^2_t$, for real GDP, $\Delta y^1_t$. Compared to table 1, we see a slightly higher $p_{22}$, implying a longer duration for recessions estimated from GDI, and we see a lower conditional variance for GDI, more pronounced in the last set of estimates. Figure 2 repeats the information in Figure 1, with the latest probabilities of recession estimated from GDI overlaid in solid blue. The GDI probabilities clearly perform better than the GDP probabilities at the NBER-defined start of recessions; at the starting point of the
1980, 1981-82, 1990-91, and 2001 recessions, these GDI probabilities were 78%, 44%, 72%, and 70%, respectively, each higher than the corresponding GDP probability. Some other notable features of this figure - the 1979 spike in the GDP and GDI probabilities, and the 2003 spike in the GDI probability, will be discussed later.

We next consider the bivariate generalization of the model, using both GDP and GDI to estimate probabilities of recession; see the FEDS working paper version of this paper, Nalewaik (2007), for details on the model. Figure 3 plots, in solid black, the real time probabilities of recession for the latest quarter from the bivariate model, along with the univariate GDP probabilities in dashed red. This figure looks very similar to Figure 2, and Figure 4 shows why: the bivariate model produces probabilities that are almost identical to the univariate GDI probabilities. In the bivariate model, GDI dominates GDP, largely determining the probabilities of recession. Table 3, showing the parameter estimates for the bivariate model from the first and last “final” current quarterly release we consider, tells us why. A greater spread between conditional means and smaller conditional variance increases the weight a variable receives in determining the probabilities of recession and expansion, and table 3 shows that real GDI growth beats real GDP growth on both counts.

One might want to account for the Great Moderation explicitly in the parameter estimates; table 4 shows estimates of the bivariate model with a 1984Q3 break in the variance parameters, while table 5 shows estimates with breaks in the conditional means as well. Figure 5 plots the probabilities of recession for the latest quarter from each “final” current quarterly release from 1999Q3 to 2005Q3, along with the probabilities without the breaks. Given the tiny post-break conditional variances, The model confidently calls the start of the recession in 2000Q4, but given the tiny post-break conditional variances, the small post-1984 sample, and concerns about over-fitting, this model should be used
Comparison with Smoothed Probabilities

Figure 6 shows smoothed probabilities, computed using the algorithm in Kim (1994), from the last time series used in the paper, the series available at the time of the 2005Q3 “final” current quarterly data release. While this figure highlights the differences between the smoothed probabilities from the bivariate model and univariate GDP model, it also highlights differences between the smoothed probabilities from the Markov switching models and the NBER recession dates. The bivariate model in particular has increasingly come to define its low-growth state as a period of mean growth around zero, including not only several quarters of negative growth strung together, but also periods of slow or around zero growth that have occurred around recent contractions. For the 1980 recession, this slow growth period includes most of 1979; for the 1990-1991 recession, this slow growth period includes most of 1989 and the early part of 1990; and for the 2001 recession, this slow growth period includes 2002 and part of 2003. The GDP-based smoothed probabilities are generally closer to the NBER dates, but the movement towards a more expansive definition of the low-growth phase is evident in the univariate GDP model as well.

So we have three candidate benchmarks to which we may compare our real time estimated probabilities of recession: the NBER’s dates, the latest smoothed univariate GDP probabilities, and the latest smoothed bivariate probabilities. However, no matter what benchmark one uses, the probabilities from the univariate GDP model, estimated using real time data, do not rise fast enough at the start of recessions or low-growth states, a problem that is alleviated by incorporating the real time information in GDI through the bivariate model. This conclusion holds when one takes the start dates
defined by the NBER as the benchmark, and Figure 6 shows that it must hold taking the smoothed probabilities as the benchmark as well, since they rise above 50% before the NBER start date for almost every recession. The only exception is the 1981-82 recession, where the smoothed bivariate probabilities suggest that the recession started in 1981Q4 rather than 1981Q3.5

3 Real Time Probabilities: A closer look around Recession Start Dates

While the behavior of GDI largely determines probabilities of recession in the bivariate model, there are good reasons to avoid dropping GDP from the model altogether. For one, the latest quarter of GDI is not available with the BEA’s “advance” estimates for that quarter released about one month after the quarter ends, and it is not always available with the BEA’s “preliminary” release about a month later. Instead of waiting to update the probabilities with information on quarter $T$ until the “final” release three months after the quarter ends, we can instead estimate the bivariate model parameters with data through quarter $T-1$, and employ a two stage updating procedure for inferences about $T$. While time $T$ GDI is unavailable, we update the probabilities using the marginal density of time $T$ GDP growth, later updating in a second stage when time $T$ GDI becomes available using its density conditional on GDP; see Nalewaik (2007) for details. Using this procedure, we can produce updated probabilities of recession from the bivariate model at each BEA release date.

5In real time, the Markov switching models with GDP, GDI, or both breached the 50% threshold in 1981Q4, agreeing with the smoothed probabilities.
The subsections that follow undertake a closer examination of probabilities of recession at interesting NIPA release dates, examining more than just the latest probability. The probabilities are smoothed with the data available in real time; for example the solid line in the top panel of Figure 7 shows the contemporaneous probability of recession for 1979Q3, computed from the time series available at the “final” 1979Q3 NIPA release, along with probabilities from prior quarters back to 1978Q3, with those probabilities smoothed with respect to that “final” 1979Q3 time series.

**The 1980 recession**

Figure 7 starts with data from the “final” 1979Q1 GNP release. The probabilities are low here despite the weak GNI and GNP growth rates of 1.4% and 0.8%, since they follow strong 6.1% and 6.9% growth in 1978Q4. However the “advance” 1979Q2 reading of -3.3% GNP growth causes the probability to shoot up to almost 90%, and it remains high with the “preliminary” 1979Q2 estimates of -2.4% GNP and -2.2% GNI growth. The smoothed probabilities in Figure 6 suggest with high probability that the low-growth phase started here.

The last half of 1979 brings weak to moderate readings on growth, with “final” estimates for 1979Q3 GNP and GNI growth at 3.1% and 1.5% (smoothed probabilities shown as the solid line in the top panel), respectively, and “final” estimates for 1979Q4 GNP and GNI growth at 2.0% and 2.7%. These growth rates damp down the real time estimated probability to below 50% (see the dotted-dashed line in the bottom panel). At the NBER-defined start of the recession in 1980Q1, “advance” GNP growth is 1.1%, causing the smoothed probabilities (the dotted line in the bottom panel) to tick up. The “final” 1980Q1 release brings 1.2% and 0.7% growth for GNP and GNI; these growth rates are low enough, combined with the history of weak growth in 1979,
to bring the 1980Q1 probability of recession well above 50%, as shown in solid line in the
bottom panel. The dashed line in the bottom panel shows that this is not the case for
probabilities computed from GNP alone: while the high probability in 1979Q2 remains,
the GNP-based probabilities show about a 50% probability that the economy has passed
through that period of weakness by 1980Q1, a notion that large negative growth in
1980Q2 dispels. The additional information provided by GNI, its slightly weaker history
post-1979Q2, allows the bivariate model to provide a clear signal in 1980Q1, as opposed
to the ambiguous signal provided by the univariate GNP model.

**The 1990-1991 recession**

Figure 8 shows smoothed probabilities around the start of the 1990-91 recession. The
economy seems to be sailing along smoothly until the annual revision that comes with
the 1990Q2 data: the four quarterly GNP growth rates in 1989 (in chronological order)
are revised down from 3.7%, 2.5%, 3.0% and 1.1% to 3.6%, 1.6%, 1.7% and 0.3%, an
average downward revision of about three quarters of a percentage point. The quarterly
growth rates of GNI are revised down even more, from 4.3%, 2.0%, 3.5% and 1.4% to
3.4%, 0.9%, 1.4% and -0.7%, an average downward revision of about one and a half
percentage points. With the revisions, the probabilities from the model indicate quite
clearly that economy was in recession or its low-growth phase in the second half of 1989
(see the dotted, dashed, and solid lines in the top panel). Positive growth from 1990Q1
to 1990Q3 brings down the probabilities in those quarters, but the growth rates are low
enough to keep the probabilities well above 50%. Negative growth in 1990Q4 solidifies
the view that the economy is in recession (see the solid line in the bottom panel).

The dashed line in the second panel of Figure 8 shows smoothed probabilities from the
univariate model with GNP alone, after the “final” 1990Q3 data release. The compara-
tively large downward revisions to GNI growth make a critical difference, especially the 2.1 percentage point downward revision that takes 1989Q4 GNI growth down to -0.7%. With GNP growth alone, the probabilities drift up but never cross the 50% threshold before the release of 1990Q4 data; these probabilities paint a much more ambiguous picture of the economy.

The 2001 recession

Figure 9 shows smoothed probabilities around the start of the 2001 recession. In 2000Q4, “advance” GDP growth (smoothed probabilities not shown) comes in at 1.4%, and is revised down to 1.1% in the “preliminary” release, which leads to an uptick in the probability of recession to about 10%. The first GDI estimate for this quarter arrives with BEA’s “final” release, and its growth of 0.3% (with GDP growth of 1.0%) bumps up the probability of recession to about 30%. In 2001Q1, when the NBER dates the start of the recession, “advance” GDP growth starts at 2.0%, again giving a probability of recession of about 30%, but when GDI growth comes in at 0.1% with the “preliminary” release, the probability of recession immediately jumps above 50%. The “final” values for 2001Q1 have GDP and GDI growth running at 1.2% and 0.0%, respectively. This case is somewhat different than the 1990 recession just discussed: it is the latest values of GDI that provide the critical information, rather than revised estimates from prior years.

The “final” values for 2001Q2 have GDP and GDI growth at 0.3% and 1.2%, respectively; the stronger value for GDI leads to a slight downtick in the probability of recession, but the probability remains above 50%. Figure 10 compares bivariate probabilities of recession computed in 2001Q1 and 2001Q2 with the corresponding univariate GDP probabilities. The differences are striking: the probabilities of recession computed
using GDP alone are much lower than those using GDP and GDI. Negative values for GDP and GDI growth in 2001Q3 (not shown) solidify the view that the economy is in recession in both models. But it was only the bivariate model that sounded clear warning bells months earlier.

It is noteworthy that the 2002 annual revision marked the beginning of a new practice for the BEA regarding the GDI growth rates: initially at the time of the “final” current quarterly release, and later accelerated to the “preliminary,” BEA began revising employee compensation and GDI for the prior quarter, replacing the current quarterly estimates of wages and salaries based on Current Employment Statistics (CES) survey data with estimates based on the Quarterly Census of Employment and Wages (QCEW), often called the ES-202 data. The QCEW data are superior to the CES data for at least two reasons: (1) the CES data are computed from samples of businesses, while the UI data are computed from the entire universe of businesses from which the CES survey is drawn, and (2) the UI data include bonuses, gains from exercising nonqualified stock options and other irregular forms of pay not captured by the CES survey. Prior to this change these data had been incorporated at annual revisions; the new practice allows BEA to produce more accurate GDI estimates sooner.

Figure 11 shows the impact of these revisions. We have, with the “final” 2002Q4 release, a downward revision to 2002Q3 GDI growth from 2.2% to 0.8%. 2002Q4 GDP and GDI growth are weak as well (“final” values at 1.4% and 2.4%), and we begin to see the probability of recession creeping back upwards. “Final” 2003Q1 GDP and GDI growth are 1.4% and 1.9% with 2002Q4 GDI revised down to 1.2%, and the probability of recession has crept back up above 50% at this point. 2003Q1 GDI growth is revised down with the 2003Q2 release, but “final” 2003Q2 GDP and GDI both come in stronger, at 3.3% and 3.1%, and the 2003Q3 estimates clearly demarcate the end of the recession
with 8.2% GDP and 5.8% GDI growth rates. With the “preliminary” 2003Q3 release came a benchmark revision that confirmed the downward revisions at the end of 2002, increasing their size to the point where the smoothed probabilities suggest the recession of 2001 did not actually end until 2003Q2, as can be seen clearly in Figure 6.

4 A True Out-of-Sample Test: the 2007-9 Recession

The work discussed thus far was done in late 2005 and early 2006, with a FEDS working paper containing the results completed in late 2006 and posted to the Federal Reserve Board web site in early 2007. Since then, as the article was moving through the submission process, the U.S. economy passed through another recession, providing a true out-of-sample test of the usefulness of the models using GDI. Interestingly, the models using GDI again recognized the onset of the low-growth state and the recession well before the univariate model using GDP. The BEA’s “final” release for 2007Q4, the quarter the NBER decided was the business cycle peak, showed GDI growth of minus 1.0%, causing the probability of recession from the model using GDP and GDI to jump up to 93%. GDP growth was also weak at 0.6% in 2007Q4, but that followed much stronger 4.9% GDP growth in 2007Q3 (compared to only 1.2% for GDI), and the probability of recession from the model with GDP alone stayed relatively low at 18%. At the BEA’s “final” releases for 2008Q1 and 2008Q2, both recession quarters, the model using GDP and GDI showed probabilities of recession of 93% and 89%, while the model with GDP alone showed probabilities of recession of only 31% and 27%. Only with the BEA’s release of data on 2008Q3 (released after the marked intensification of financial market turmoil following the bankruptcy of Lehman brothers) did the probability of recession from the model with GDP alone breach 50%; the probability was 65% after the “final”
release for 2008Q3, compared to 99% from the model with both GDP and GDI. See also Hamilton (2010), and Nalewaik (2010a, b).

5 Conclusions

This paper estimates Markov switching models for GDP and GDI in real time, showing that GDI has recognized the start of recent recessions faster than GDP. Some combination of the following two statistical facts accounts for this: (1) the deceleration of GDI growth in recessions is greater than the deceleration of GDP growth, (2) the variance of GDI conditional on the estimated state of the world is lower, so it provides a more accurate signal of the state. Overall, the results suggest that in analyzing the current state of the economy, GDI should be scrutinized closely along with GDP. The bivariate Markov switching model employed in this paper is a useful tool for summarizing the information in both estimates about the state of the economy.

Going forward, there is some reason to believe that BEA’s real time estimates of employee compensation for current quarters will be better than they have been in the past. First, BEA now incorporates what are essentially census counts of firms’ wages and salaries into its quarterly estimates with a shorter lag than it has in the past, about 5 months after the quarter closes. Second, the extrapolators BEA uses before it has access to these census counts are likely to improve as well, as BLS rolls out more comprehensive earnings measures over the next couple of years as part of its monthly Current Employment Statistics (CES) data releases. Since employee compensation makes up the bulk of GDI, these changes give further reason to study GDI carefully in analyzing the current state of the economy in the years to come.

An interesting side result of the paper is that, over the past 25 years through 2005,
the Markov switching models have gradually increased the mean growth rates of GDP and GDI in low-growth periods, to the point where they are now around zero rather than significantly negative; see also Kim and Nelson (1999b). Going hand in hand with this, these low-growth periods as defined by the switching models are now substantially longer than recessions as defined by the NBER’s peak and trough dates.

References


Table 1:
Real Time Estimates of Markov Switching Model for GDP

Data as of 1978Q1 Final Curr. Qtrly. Release

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Table 2:
Real Time Estimates of Markov Switching Model for GDI

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<td>(0.53)</td>
<td>(0.94)</td>
<td>(0.73)</td>
<td>(0.75)</td>
<td>(0.03)</td>
<td>(0.07)</td>
<td>(1.17)</td>
</tr>
</tbody>
</table>
Table 4:
Real Time Estimates of Markov Switching Model for GDP and GDI,
1984Q3 Breaks in Variance Parameters
Data as of 2005Q3 Final Curr. Qtrly. Release

<table>
<thead>
<tr>
<th></th>
<th>$\mu_{S=1}^1$</th>
<th>$\mu_{S=1}^2$</th>
<th>$\mu_{S=2}^1$</th>
<th>$\mu_{S=2}^2$</th>
<th>$(\sigma^1)^2$</th>
<th>$(\sigma^2)^2$</th>
<th>$\sigma^{12}$</th>
<th>$p_{11}$</th>
<th>$p_{22}$</th>
<th>$p_{1,\text{start}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984Q3</td>
<td>4.12</td>
<td>4.32</td>
<td>1.08</td>
<td>0.58</td>
<td>15.18</td>
<td>11.46</td>
<td>11.54</td>
<td>0.93</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.22)</td>
<td>(0.39)</td>
<td>(0.41)</td>
<td>(2.42)</td>
<td>(2.06)</td>
<td>(2.11)</td>
<td>(0.03)</td>
<td>(0.07)</td>
<td>(1.60)</td>
</tr>
<tr>
<td>Post-1984Q3</td>
<td>4.12</td>
<td>4.32</td>
<td>1.08</td>
<td>0.58</td>
<td>2.62</td>
<td>2.40</td>
<td>1.16</td>
<td>0.93</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.22)</td>
<td>(0.39)</td>
<td>(0.41)</td>
<td>(0.40)</td>
<td>(0.44)</td>
<td>(0.38)</td>
<td>(0.03)</td>
<td>(0.07)</td>
<td>(1.60)</td>
</tr>
</tbody>
</table>
Table 5:
Real Time Estimates of Markov Switching Model for GDP and GDI,
1984Q3 Breaks in both Mean and Variance Parameters
Data as of 2005Q3 Final Curr. Qtrly. Release

\[
\begin{array}{cccccccccc}
\mu_{S=1}^1 & \mu_{S=2}^1 & \mu_{S=1}^2 & \mu_{S=2}^2 & (\sigma^1)^2 & (\sigma^2)^2 & \sigma^{12} & p_{11} & p_{22} & p_{1,\text{start}} \\
\hline
\text{Pre-1984Q3} & & & & & & & & & \\
5.29 & 5.33 & -0.90 & -1.04 & 11.71 & 8.43 & 8.30 & 0.93 & 0.81 & 1.00 \\
(0.44) & (0.38) & (0.81) & (0.79) & (1.78) & (1.34) & (1.42) & (0.03) & (0.07) & (1.16) \\
\hline
\text{Post-1984Q3} & & & & & & & & & \\
3.90 & 4.11 & 1.33 & 0.78 & 2.69 & 2.43 & 1.22 & 0.93 & 0.81 & 1.00 \\
(0.24) & (0.25) & (0.42) & (0.41) & (0.48) & (0.43) & (0.37) & (0.03) & (0.07) & (1.16) \\
\end{array}
\]