Government Spending Shocks in Quarterly and Annual Time Series*

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Abstract

Government spending shocks are frequently identified in quarterly time-series data by ruling out a contemporaneous response of government spending to other macroeconomic aggregates. We provide evidence that this assumption may not be too restrictive for annual time-series data.

Keywords: Government spending shocks, Annual Data, Identification

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1 Introduction

Vector autoregressions (VAR) are by now frequently employed to study the fiscal transmission mechanism. In order to identify government spending shocks, a number of authors

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assume that there is no contemporaneous response of government spending to macroeconomic aggregates, i.e. that government spending is predetermined.\textsuperscript{1} This requires that government spending does i) neither respond automatically to the economy, ii) nor that it is adjusted in a discretionary manner within the period. The first requirement is likely to be satisfied if government spending does not include transfers, but only government consumption and investment (a commonly used definition of government spending). Whether the second requirement is satisfied depends on the extent of decision lags in the policy process and thus on the data frequency.

Blanchard and Perotti (2002) and several subsequent studies impose the identification assumption on quarterly time-series data for the U.S. and a small number of other countries, thereby ruling out a discretionary policy response to the state of the economy within the quarter. However, as long time series for non-interpolated fiscal data are often not available for most other countries at quarterly frequency, several authors impose the identification assumption at an annual frequency (see Beetsma et al. 2006, 2008, Bénétrix and Lane 2009).\textsuperscript{2} Judged a priori, the identification assumption is more compelling when imposed at quarterly frequency. While there is only one budget legislated before the beginning of the fiscal year, supplements throughout the year are always a possibility (see Perotti 2005).

However, as long time series for fiscal data are available at quarterly frequency for a number of countries, it is possible to actually test the assumption that government spending is predetermined within the year. In this paper, we suggest and perform such tests. We spell out and impose restrictions on a quarterly VAR model implied by the assumption that annual government spending is predetermined. We find that these restrictions are not rejected by the data. Also, the identified shocks and the resulting impulse response

\textsuperscript{1}This approach goes back to Blanchard and Perotti (2002). Alternative identification schemes are based on military events or sign restrictions, see, e.g. Ramey (2011) and Mountford and Uhlig (2009).

\textsuperscript{2}Recently, Ilzetzki et al. (2010) collected non-interpolated quarterly data for government spending for a sample of 44 countries. However, in most countries the time series only starts in the mid or late 1990s.
functions are very similar to those obtained from an unrestricted model.\(^3\) Finally, we compare the annualized impulse responses and shocks to those obtained from a model estimated on annual data and find a high degree of conformity.

2 A structural VAR model

In this section, we devise a simple test of the predeterminedness of annual government spending, conditional on it being predetermined at the quarterly level. We proceed in three steps. First, we specify a data generating process operating at quarterly frequency where government spending is predetermined. Second, we derive a representation of the annual time series generated by the quarterly model. Third, we spell out restrictions on the quarterly model under which annual government spending is predetermined. In a nutshell, we test whether government spending responds to the other variables included in the VAR model only after a delay of four quarters.

2.1 Data generating process

Consider a vector of endogenous variables, \(y_t = \begin{bmatrix} g_t & x_t' \end{bmatrix}'\), where \(g_t\) denotes government spending and \(x_t\) denotes a \(n \times 1\) vector of additional variables. The data are sampled at quarterly frequency through the structural VAR(4) model:

\[
A^{(0)}y_t = A^{(1)}y_{t-1} + A^{(2)}y_{t-2} + A^{(3)}y_{t-3} + A^{(4)}y_{t-4} + \varepsilon_t,
\]

(1)

\(^3\)In related work, Beetsma et al. (2009) suggest an alternative approach to assess the validity of identification restrictions in a low-frequency VAR model. Specifically, given estimates for a high frequency model, they compute estimates for the implied within-period change of variables in the low-frequency model. It is then possible to test whether certain restrictions on the low-frequency model are satisfied. In contrast, we proceed by specifying and testing restrictions on the quarterly model which ensure that government spending is predetermined at annual frequency. On the basis of their test and a panel of European data, Beetsma et al. (2009) cannot reject the hypothesis that government spending does not respond to economic activity within the year.
where $\varepsilon_t = \begin{bmatrix} \varepsilon^g_t & \varepsilon^x_t \end{bmatrix}'$ is a vector of mutually uncorrelated structural shocks.

We assume that government spending is predetermined, i.e. that the non-fiscal entries of the first row of $A^{(0)}$ are zero. Further assuming that $A^{(0)}$ is lower triangular allows recursive estimation of model (1) by OLS without loss of generality, as long as the interest is in identifying government spending shocks and no structural interpretation is given to $\varepsilon^x_t$ (see Christiano et al. 1999). For future reference, it is convenient to partition the $A^{(i)}, i = 0 \ldots 4$, while appropriately restricting the impact matrix $A^{(0)}$:

$$
A^{(0)} = \begin{bmatrix}
1 & 0 \\
\alpha^{(0)}_{nxg} & Q
\end{bmatrix}_{n \times n},
-A^{(i)} = \begin{bmatrix}
\alpha^{(i)}_{ggx} & \alpha^{(i)}_{gxx} \\
\alpha^{(i)}_{xgx} & \alpha^{(i)}_{xx}
\end{bmatrix}_{1 \times n}, i = 1 \ldots 4,
$$

where $Q$ is a lower-triangular matrix with 1’s on its main diagonal.

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2.2 Aggregation

We are now interested in how data generated by the process (1) aggregates into data sampled at annual frequency. Following Lütkepohl (2006, p. 441), we obtain the system:

\[
\begin{bmatrix}
A^{(0)} & 0 & 0 & 0 \\
-A^{(1)} & A^{(0)} & 0 & 0 \\
-A^{(2)} & -A^{(1)} & A^{(0)} & 0 \\
-A^{(3)} & -A^{(2)} & -A^{(1)} & A^{(0)}
\end{bmatrix}
\begin{bmatrix}
y_{4(\tau)} \\
y_{4(\tau-1)} \\
y_{4(\tau-2)} \\
y_{4(\tau-3)}
\end{bmatrix}
= 
\begin{bmatrix}
y_{4(\tau-1)+1} \\
y_{4(\tau-1)+2} \\
y_{4(\tau-1)+3} \\
y_{4(\tau)}
\end{bmatrix}
\]

\hat{\eta}_{\tau} 

It is convenient to reshuffle the variables in \( \eta_{\tau} \). Let

\[ \hat{\eta}_{\tau} = 
\begin{bmatrix}
g_{4(\tau-1)+1} & g_{4(\tau-1)+2} & g_{4(\tau-1)+3} & g_{4\tau} & x'_{4(\tau-1)+1} & x'_{4(\tau-1)+2} & x'_{4(\tau-1)+3} & x'_{4\tau}
\end{bmatrix}^{'}
\]

and use a ‘tilde’ to denote the appropriately reshuffled counterparts of the matrices \( B^{(i)} \), \( i = 0, 1 \), and the shock vector \( u_{\tau} \). One may then write (3) as follows:

\[
\tilde{B}^{(0)} \hat{\eta}_{\tau} = \tilde{B}^{(1)} \hat{\eta}_{\tau-1} + \tilde{u}_{\tau}.
\]
Aggregating a quarterly time series into an annual time series, denoted by an ‘a’-superscript, corresponds to the following linear transformation

\[ y^a_t = \begin{bmatrix} g^a_t \\ x^a_t \end{bmatrix} = F \tilde{\eta}_t, \text{ where } F = \begin{bmatrix} \iota & 0 \\ 0 & K \end{bmatrix}, \]

Here, \( \iota \) is a row vector of ones and \( K \) an appropriately defined matrix. Applying \( F \) to the reduced form system of (4) gives

\[ y^a_t = FC\tilde{B}^{(1)}\tilde{\eta}_{t-1} + FC\tilde{u}_t, \tag{5} \]

where \( C^{-1} = \tilde{B}^{(0)} \).

2.3 When is annual spending predetermined?

Relationship (5) maps quarterly data into annual observations. We spell out sufficient conditions on the underlying quarterly model such that annual government spending is predetermined, i.e., we require that the linear mapping \( FC \) in (5) excludes non-fiscal innovations to have an impact on \( g^a_t \). We can then state the following proposition.

**Proposition 1.** Annual government spending is predetermined with respect to \( x^a_t \) if the following linear restrictions are satisfied:

\[ a_{g^a_x}^{(1)}, a_{g^a_x}^{(2)}, a_{g^a_x}^{(3)} = 0, \tag{6} \]

where \( a_{g^a_x}^{(i)}, i = 1, 2, 3, \) is defined in (2).

**Proof.** See appendix A.

These restrictions only concern the first equation of our quarterly model (1), allowing us...
to test the joint null hypothesis

$$H_0 : \ a_{g1} = a_{g2} = a_{g3} = 0$$

(7)

in the single OLS regression

$$g_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} + \varepsilon_t^g,$$

(8)

where $\beta_i = \begin{bmatrix} a_{gg}^{(i)} & a_{g}^{(i)} \end{bmatrix}$, $i = 1, \ldots, 4$.

We use two approaches to test the null hypothesis. A likelihood-ratio test is given by

$$LR = T (\ln \sigma_r^2 - \ln \sigma_u^2),$$

(9)

where $\sigma_r^2$ is the residual variance of regression (8) when the restrictions are imposed, $\sigma_u^2$ is the residual variance of the unrestricted regression, and $T$ is the number of observations. For the likelihood-ratio statistic, we assume joint normality of the disturbances. As an alternative approach, which is robust to variations in the underlying distribution, we consider the Wald statistic. Let $\beta = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{bmatrix}'$ and $\hat{\beta}$ its corresponding sample estimator. The null hypothesis that a subvector $\beta_0$ of $\beta$ is equal to zero can then be tested by the Wald statistic

$$W = \hat{\beta}_0 V_0^{-1} \hat{\beta}_0,$$

(10)

where $V_0$ denotes the submatrix of the estimated covariance matrix $V$ corresponding to $\hat{\beta}_0$. Both the likelihood ratio and the Wald statistic have a limiting $\chi^2(j)$-distribution where $j$ is the number of zero restrictions imposed on $\beta$. 

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3 Time series evidence

In this section, we proceed in three steps. First, we briefly discuss our data and empirical specification. Second, we use quarterly data to test whether annual government spending is predetermined. Third, we compare results obtained on the basis of annual and quarterly data.

3.1 Data and specification

In the following, we estimate a VAR model on time-series data for Australia, Canada, the U.K., and the U.S., as long time series of non-interpolated fiscal data are available for these countries.\(^4\) We use quarterly data to estimate equation (1), where the model also contains a constant and a linear time trend. In the baseline case, the vector of endogenous variables contains government spending, GDP, and private consumption. Given the limited number of annual observations, this parsimonious specification allows a comparison with the results obtained from a VAR model estimated on annual data for the same sample period. To explore the robustness of our results, we also consider, for the U.S., a 7-variable VAR model for quarterly data, augmenting the baseline VAR with net tax revenues, private investment, inflation, and the 3-month T-bill rate, thereby following Perotti’s (2007) specification closely.\(^5\)

\(^4\)In each case we consider data up to 2007Q4, while starting dates for the dependent variable differ due to data availability in Australia (1960Q3), Canada (1962Q1), U.K. (1964Q1), and the U.S. (1954Q1). Details on the data are provided in an online appendix available on the authors' webpages.

\(^5\)In the small VAR, identification is achieved by excluding a response of government spending within the quarter. In the large VAR, we follow Perotti (2005) and assume that the price elasticity of real government spending is \(-0.5\). In this case, we use an inflation adjusted measure of government spending when estimating the equation for spending and also in the testing equation (8). In addition, we use instrumental variables when estimating the VAR recursively.
3.2 Is annual spending predetermined?

In estimating the VAR model on quarterly data, we test restrictions (6). Results reported in Table 1 show that we cannot reject the null hypothesis (7) for any of the countries which we consider. This finding is consistent with the hypothesis that annual government spending is predetermined.

Table 1 and Figure 1 about here

For quarterly U.S. data, Figure 1 presents the impulse response functions to an increase in government spending by one percent of GDP. The upper row shows results for the baseline 3-variable VAR model, the lower row shows the results for government spending, output and consumption obtained from the 7-variable VAR model. Importantly, in all panels we show impulse responses of the restricted (dashed line) and the unrestricted model (solid line). There is, however, hardly any difference across these responses—suggesting, in line with the results reported in table 1, that the restrictions (6) are easily tolerated by the data. Finally, we note that results for the trivariate VAR and the 7-Variable specification are fairly similar (see also Perotti 2007). Results for Australia, Canada and the U.K. are also very similar in that the restricted and the unrestricted models give rise to impulse response functions which are virtually identical.\(^6\) In the following, we focus on the results from the trivariate VAR model estimated on U.S. data.

3.3 Spending shocks in quarterly and annual time-series

We now consider results obtained from estimating a trivariate VAR model on annual data, while allowing for two lags.\(^7\) Given the results of the previous section, we impose the restric-

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\(^6\) However, for these countries we find the responses of output and consumption more contained relative to the U.S. They are displayed, together with additional results for the next subsection, in the appendix available on the authors’ webpages.

\(^7\) The Schwarz Information Criterion proposes two lags while Akaike and a recursive LR test propose three lags. We use the more parsimonious specification as it is closer to the quarterly VAR(4) model.
tion that annual government spending is predetermined to identify government spending shocks.

Figure 2 about here

Figure 2 reports the responses of government spending, output, and private consumption to an exogenous increase in government spending by one percent of GDP obtained from the VAR model estimated on annual data (solid line with squares). Qualitatively, results are similar to those obtained under the quarterly model. For a systematic comparison, we annualize the responses of the unrestricted quarterly baseline model and plot them in the panels of figure 2 (dashed-dotted line with circles). Clearly, while some differences can be observed, the annualized responses obtained from the quarterly model are fairly close to those obtained from the annual model.\(^8\)

Finally, we also compare the identified government spending shocks obtained from the annual model with the annualized shocks obtained from the unrestricted quarterly model. The shock series are plotted in figure 3, showing a high degree of correlation.

Figure 3 about here

4 Conclusion

Several authors have turned to VAR models to investigate the fiscal transmission mechanism and identified government spending shocks by ruling out a contemporaneous response of government spending to the state of the economy. This assumption is fairly plausible at quarterly frequency, because decision lags make it difficult for policy makers to engineer discretionary fiscal measures within the quarter.

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\(^8\)The annualized responses of the quarterly model tend to peak earlier than the responses of the annual model. This shift to the left disappears, once eight lags are included in the quarterly model.
For lack of long time series of quarterly data, several studies use annual data and impose the assumption that annual government spending is predetermined as well. This assumption is more restrictive, given that supplements to the annual budget may be legislated throughout the year. However, as annual and quarterly fiscal data are available for some countries, it is possible to test whether annual government spending is predetermined conditional on it being predetermined at quarterly frequency.

We perform several tests and provide evidence that annual government spending is indeed predetermined. Thus, in case sufficiently long time series of fiscal data are not available at quarterly frequency, our results provide support for resorting to annual data and, in order to identify government spending shocks, to assume that annual government spending is predetermined. Yet, in concluding, we acknowledge that this identification scheme remains subject to the caveat that innovations to government spending may to some extent be anticipated (see Ramey 2011) or, more generally, that, as fiscal innovations are concerned, the information sets of private agents and econometricians are not fully aligned (see Leeper et al. 2009).

Literature Cited


A Proof of Proposition 1

Proof. Given the process defined by equation (5), \( g^a_i \) is predetermined relative to \( x^a_i \) if FC is lower-triangular. Assuming for simplicity that \( n = 1 \), we have

\[
FC = \begin{bmatrix}
\sum_{j=1}^{4} c_{j1} & \sum_{j=1}^{4} c_{j2} & \sum_{j=1}^{4} c_{j3} & \sum_{j=1}^{4} c_{j4} & \sum_{j=1}^{4} c_{j5} & \sum_{j=1}^{4} c_{j6} & \sum_{j=1}^{4} c_{j7} & \sum_{j=1}^{4} c_{j8} \\
\sum_{j=5}^{8} c_{j1} & \sum_{j=5}^{8} c_{j2} & \sum_{j=5}^{8} c_{j3} & \sum_{j=5}^{8} c_{j4} & \sum_{j=5}^{8} c_{j5} & \sum_{j=5}^{8} c_{j6} & \sum_{j=5}^{8} c_{j7} & \sum_{j=5}^{8} c_{j8}
\end{bmatrix}.
\]

A sufficient condition for FC to be lower is that \( C \) is lower triangular:

\[
FC = \begin{bmatrix}
\sum_{j=1}^{4} c_{j1} & \sum_{j=1}^{4} c_{j2} & \sum_{j=1}^{4} c_{j3} & \sum_{j=1}^{4} c_{j4} & 0 & 0 & 0 & 0 \\
\sum_{j=5}^{8} c_{j1} & \sum_{j=5}^{8} c_{j2} & \sum_{j=5}^{8} c_{j3} & \sum_{j=5}^{8} c_{j4} & 0 & 0 & 0 & 0
\end{bmatrix}.
\]

As \( C^{-1} = \tilde{B}^{(0)} \), \( C \) is lower triangular if \( \tilde{B}^{(0)} \) is lower triangular. Given the definition of \( \tilde{B}^{(0)} \),

\[
\tilde{B}^{(0)} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
(1) & 1 & 0 & 0 & (1) & 0 & 0 & 0 \\
(2) & (1) & 1 & 0 & (2) & (1) & 0 & 0 \\
(3) & (2) & (1) & 1 & (3) & (2) & (1) & 0 \\
(0) & 0 & 0 & 0 & Q & 0 & 0 & 0 \\
(1) & (0) & 0 & 0 & (1) & Q & 0 & 0 \\
(2) & (1) & 0 & 0 & (2) & (1) & Q & 0 \\
(3) & (2) & (1) & 0 & (3) & (2) & (1) & Q
\end{bmatrix},
\]

the latter is true if \( a_{gx}^{(1)} = a_{gx}^{(2)} = a_{gx}^{(3)} = 0. \)
Table 1: Test Statistics

<table>
<thead>
<tr>
<th></th>
<th>LR-statistic</th>
<th>Wald-statistic</th>
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</thead>
<tbody>
<tr>
<td><strong>Australia</strong></td>
<td>8.59</td>
<td>8.79</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.19)</td>
</tr>
<tr>
<td><strong>Canada</strong></td>
<td>4.65</td>
<td>4.71</td>
</tr>
<tr>
<td></td>
<td>(0.59)</td>
<td>(0.58)</td>
</tr>
<tr>
<td><strong>U.K.</strong></td>
<td>7.46</td>
<td>7.62</td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
<td>(0.27)</td>
</tr>
<tr>
<td><strong>U.S.</strong></td>
<td>3.13</td>
<td>3.15</td>
</tr>
<tr>
<td></td>
<td>(0.79)</td>
<td>(0.79)</td>
</tr>
<tr>
<td>7-Variable VAR</td>
<td>18.75</td>
<td>19.59</td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.36)</td>
</tr>
</tbody>
</table>

*Notes:* values are test statistics, p-values are given in parentheses. Upper panel displays results for trivariate VAR model (test distribution: $\chi^2(6)$), lower panel displays results for 7-variable VAR for U.S. (test distribution: $\chi^2(18)$).
Figure 1: Effect of government spending shock in the U.S. (quarterly data). Notes: Impulse responses to exogenous increase in real government spending by one percent of GDP. Solid line: unrestricted baseline model; shaded areas: bootstrapped 90 percent confidence intervals; dashed line: restricted baseline model. Vertical axes indicate deviations from unshocked path in percent of GDP. Horizontal axes indicate quarters. Results for Australia, Canada and the U.K. are displayed in the appendix available on the authors’ webpages.
Figure 2: Effect of government spending shock in the U.S. (annual vs. annualized responses). Notes: Impulse responses to exogenous increase in real government spending by one percent of GDP. Solid line with squares: annual VAR model; shaded areas: bootstrapped 90 percent confidence intervals; dashed-dotted line with circles: annualized impulse responses from unrestricted quarterly trivariate model. Vertical axes indicate deviations from unshocked path in percent of GDP. Horizontal axes indicate years. Results for Australia, Canada and the U.K. are displayed in the appendix available on the authors’ webpages.
Figure 3: Annual vs. annualized shocks in the U.S. Notes: Solid line with squares: shocks identified in annual VAR model; dashed-dotted line with circles: annualized shocks of trivariate VAR model. Results for Australia, Canada and the U.K. are displayed in the appendix available on the authors’ webpages.