Taxation and Income Distribution Dynamics in a Neoclassical Growth Model∗

Cecilia García-Peñalosa
Aix-Marseille University

Stephen J. Turnovsky
University of Washington, Seattle

Revised version
May 2011

Abstract: We examine how changes in tax policies affect the dynamics of the distributions of wealth and income in a Ramsey model in which agents differ in their initial capital endowments. The endogeneity of the labor supply plays a crucial role, as tax changes that affect hours of work will affect the distribution of wealth and income, reinforcing or offsetting the direct redistributive impact of taxes. We consider different ways of financing government expenditure and find that policies that reduce the labor supply are associated with lower output but also with a more equal distribution of after-tax income. We illustrate these effects by examining the impact of recent tax changes observed in the US and in European economies.

JEL Classification Numbers: D31, O41

Key words: taxation; wealth distribution; income distribution; endogenous labor supply; transitional dynamics.

∗The paper has benefited from seminar presentations at the Free University of Berlin, at GREQAM, and at Washington State University, as well as presentations at the conference “Growth with Heterogeneous Agents: Causes and Effects of Inequality”, Marseille, June 2008, the 14th Conference on Computation in Economics and Finance, Paris, June 2008 the 3rd Workshop on Macroeconomic Dynamics, held in Melbourne, July 2008, the 2009 PET conference in Galway. Comments received at these various presentations are gratefully acknowledged. In particular, we thank Jess Benhabib, Julio Davila, and Roger Farmer for their comments. The paper has also benefited from the constructive suggestions of two referees and the journal’s editor. García-Peñalosa is a CNRS researcher at GREQAM (Aix-Marseille University), and would like to acknowledge the support received from the French National Research Agency Grant ANR-08-BLAN-0245-01. Turnovsky’s research was supported in part by the Castor endowment at the University of Washington.
1. Introduction

The role of taxation in the neoclassical growth model has been extensively studied, and the impact of different taxes on both the long-run equilibrium and transitional dynamics is well documented. However, the implications of the tax structure for the distributions of income and wealth have received much less attention. Two notable exceptions are Krusell, Quadrini, and Rios-Rull (1996) and Correia (1999). Krusell, Quadrini, and Rios-Rull examine the efficiency and distributional effects of switching from an income tax to a consumption tax. Correia proposes a methodology to rank alternative aggregate equilibria in terms of their distributional implications by examining the effect on factor prices. In this paper we examine the distributional effects of taxes levied on capital income, labor income, and consumption. We characterize both the steady-state distributions of income and wealth, as well as their respective transitional dynamics, and discuss how these responses depend upon the allocation of the tax revenues raised. This is important, since as we shall demonstrate below, fiscal policy typically involves sharp tradeoffs between its effects on the level of activity and its distributional consequences.

In García-Peñalosa and Turnovsky (2007) we discuss the tradeoffs between growth and distribution generated by tax policies using a simple endogenous growth model. The AK-technology employed has the key feature characteristic of that class of models: there are no transitional dynamics, and hence the distribution of capital remains constant over time. As a result, the full impact of any policy shock on the distribution of income occurs immediately and only through changes in factor prices and the aggregate labor supply. This raises the question of the effect of policy on income distribution under more general technological conditions, which inevitably cause transitional adjustments in the economy, resulting in the distribution of wealth evolving over time.

The importance of changes in relative wealth for income dynamics was established in our previous work, Turnovsky and García-Peñalosa (2008). There we show that the distributional effects of improvements in technology were driven by a combination of changes in factor prices and

---

1 See, for example, Judd (1985), Chamley (1986), Lucas (1990), Stokey and Rebelo (1991), Ladrón-de-Guevara et al. (2002), and Turnovsky (2004).
changes in the distribution of wealth, which tended to move in opposite directions during the transition, making their overall effect difficult to assess. Moreover, the short-run and long-run changes in the distribution of income were in opposite directions, as the initial response in inequality to a productivity shock was then partially or totally offset by changes in the distribution of wealth during the transition to the new steady state. Hence, our previous work raises two questions concerning the impact of taxation on income distribution in a neoclassical growth model: to what extent does a conflict exist between output and distribution (as in the AK model we have previously examined), and whether the trade-off between the short- and the long-run responses of inequality caused by a productivity increase also occurs as a result of tax changes.

In this paper we employ the one-sector Ramsey model, developed in Turnovsky and García-Peñalosa (2008), in which agents differ in their initial endowments of capital (wealth). Representing preferences by a utility function that is homogeneous in consumption and leisure allows aggregation as in Gorman (1953) or Eisenberg (1961), and generates a representative-consumer characterization of the macroeconomic equilibrium. This homogeneity assumption – which in any event is the dominant one in contemporary growth theory – yields a substantial payoff in terms of analytical tractability, enabling us to examine distributional issues sequentially. First, the dynamics of the aggregate stock of capital and labor supply are jointly determined, independently of distributional considerations. The distributions of wealth and income and their dynamics are then characterized in terms of the aggregate magnitudes.

In this context, there are three mechanisms that we must consider to understand the distributional implications of a fiscal policy change. First, a policy shock will in general cause a change in the steady-state capital stock. During the transition to the new steady state, agents with different initial wealth levels accumulate capital at different rates so that wealth inequality may either increase or decrease over time. In Turnovsky and García-Peñalosa (2008) we showed that this aspect was important when considering the distributional effect of an increase in productivity.

---

2 There are of course many potential sources of heterogeneity giving rise to distributional issues, but differential initial wealth endowments seems to us to be arguably one of the most significant. For evidence of the importance of inherited wealth see Piketty (2010).

3 Without the homogeneity assumption, aggregate behavior and distribution become simultaneously determined and analysis of the transitional dynamics becomes intractable.
Second, changes in taxes induce labor supply responses that will have distributional implications. As in our previous work, we derive a negative relationship between agents’ relative wealth (capital) and their relative allocation of time between work and leisure. Wealthier agents have a lower marginal utility of wealth, and hence choose to consume more of all goods, including leisure, thus reducing their labor supply. Third, the dynamics of the response of income inequality (both before-tax and after-tax) are driven by two effects: the initial jump in leisure - which will affect relative labor incomes- and the continuous changes in the distributions of capital and factor prices that occur during the transition to the new steady state. These two responses may move in opposite directions, implying non-monotonic responses to policy changes.

We characterize the time paths of the distributions of wealth and income, as well as their steady-state distributions, and examine their responses to changes in tax rates. In general we show how the evolution of wealth inequality is driven by the dynamics of aggregate labor supply (leisure) through its response to the accumulation of aggregate capital stock. In particular, economy-wide accumulation of capital is associated with a reduction in wealth inequality, as wealthier agents, choosing to enjoy more leisure, accumulate capital at slower rates. This in turn drives the evolution of pre-tax income distribution through its impacts on the relative capital income and relative labor income. In addition to being driven by these same determinants, post-tax income inequality is also dependent on the direct redistributive effects associated with taxes on labor and capital income. To illustrate these channels we analyze the comparative effects of raising alternative tax rates to finance a given increase in government spending on goods, on the one hand, versus financing transfers, on the other. In all cases we find that the income distribution effects dominate the wealth accumulation effects, which are in general very weak and entail only minor changes in the distribution of wealth. One consequence of this is that the effects of alternative income taxes on both pre-tax and post-tax income distribution depend critically upon how the resulting revenue is spent.

---

5 There is substantial empirical evidence in support of this negative relationship between wealth and labor supply. Holtz-Eakin, Joulfaian, and Rosen (1993) provided evidence to suggest that large inheritances decrease labor participation. Cheng and French (2000) and Coronado and Perozek (2003) use data from the stock market boom of the 1990s to study the effects of wealth on labor supply and retirement, finding a substantial negative effect on labor participation. Algan, Chéron, Haireault, and Langot (2003) employ French data to analyze the effect of wealth on labor market transitions, and find a significant wealth effect on the extensive margin of labor supply.
As a specific example, consider the impact of a higher tax on labor income. The aggregate effect is to reduce labor supply, the capital stock, and total output and therefore to increase wealth inequality. While it reduces the aggregate labor supply, the tax on labor income reduces that of the capital-rich by more than that of the capital-poor. This results in a more dispersed distribution of labor incomes, and since these are negatively correlated with capital endowments, it tends to decrease pre-tax income inequality. But in addition, the direct redistributive effect from a higher tax on labor income tends to increase post-tax income inequality for a given degree of pre-tax income inequality. If the revenues are rebated neutrally, post-tax income inequality still declines, but if they are spent on a utility-enhancing public good, this effect may dominate, and post-tax income inequality may actually rise. The responses of pre-tax and post-tax income inequality are reversed in the case of a tax on capital income.

Our paper contributes to the recent literature characterizing distributional dynamics in growth models, an issue first examined by Stiglitz (1969) using a form of the Solow model. One approach has considered economies with ex-ante identical agents and uninsurable, idiosyncratic shocks. An alternative approach has been to assume that agents differ in their initial capital endowments, as we do in our model. We follow Caselli and Ventura (2000), who study the dynamics of wealth and income distribution in the Ramsey model, though they restrict their analysis to exogenous labor supply, and do not consider the impact of tax policies. Closely related is Benhabib and Bisin (2008), who also examine the distributive impact of taxes and derive explicit expressions for the distribution of wealth. The analytical frameworks are, however, rather different. They consider an overlapping generations setup in which agents differ in their degree of altruism, and focus on the role of the intergenerational transmissions of wealth and state taxation in generating an empirically plausible distribution of wealth. In contrast, we abstract from these aspects by considering infinitely-lived agents, and center our analysis around the endogeneity of the labor supply, the way in which it is affected by taxation, and its impact on both wealth and income distributional dynamics.

---

6See Bertola, Foellmi, and Zweimüller (2006) for a survey.
Following this introduction, Section 2 describes the economy and derives the macroeconomic equilibrium. Section 3 characterizes the distributions of wealth and income and derives the main analytical results. Section 4 derives the effects of changes in tax rates on the long-run distributions of wealth and income, which are then illustrated in Section 5 with a number of numerical examples. Section 6 concludes, while insofar as possible technical details are relegated to an Appendix.

2. The Analytical Framework

The analytical framework we employ is developed by Turnovsky and García-Peñalosa (2008) where it is discussed in greater detail. Aggregate output is produced by a single representative firm in accordance with the neoclassical production function

\[ Y = F(K, L) \quad F_L > 0, F_K > 0, F_{LL} < 0, F_{KK} < 0, F_{LK} > 0 \quad (1) \]

where, \( K \), \( L \) and \( Y \) denote the per capita stock of capital, labor supply and output. The wage rate, \( w \), and the return to capital, \( r \), are determined by the marginal physical products of labor and capital,

\[ w(K, L) = F_L(K, L) \quad w_K = F_{LK} > 0, \quad w_L = F_{LL} < 0 \quad (2a) \]

\[ r(K, L) = F_K(K, L) \quad r_K = F_{KK} < 0, \quad r_L = F_{KL} > 0 \quad (2b) \]

2.1 Heterogeneous consumers

We assume a constant population, \( N \). Individual \( i \) owns \( K_i(t) \) units of capital at time \( t \), so that the total amount of capital in the economy at time \( t \) is

\[ K^T(t) = \int_0^N K_i(t) \, di \]

and the average per capita amount of capital is

\[ K(t) = \frac{1}{N} \int_0^N K_i(t) \, di . \]

We define the relative share of capital owned by agent \( i \) to be \( k_i(t) = K_i(t) / K(t) \) the mean of which

---

9 We should emphasize that Turnovsky and García-Peñalosa (2008) focuses on very different issues, being concerned with structural aspects such as the role of flexible labor supply and indeed abstracts from tax issues being addressed here.
is the initial (given) relative endowment of agent \( i \) is \( k_{i0} \), the initial (given) standard deviation of which is \( \sigma_{k_i} \). To ensure that all individuals supply some labor we further suppose that initial relative capital is bounded above by \( \bar{k} \).

Each individual is endowed with a unit of time that can be allocated either to leisure, \( l_i \), or to work, \( 1-l_i \equiv L_i \), so that average (aggregate) labor and leisure can be expressed as 
\[
L = 1 - l = \left(1/N\right) \int_0^N (1-l_i(t)) \, dt.
\]
The agent maximizes lifetime utility, assumed to be a function of consumption, the amount of leisure time, and government expenditure, \( G \) (taken as given), in accordance with the isoelastic utility function
\[
\max \int_0^1 \frac{1}{\gamma} \left(C_i(t)l_i^\gamma G^\theta e^{-\beta t}\right) \, dt, \quad \text{with} \quad -\infty < \gamma < 1, \eta > 0, 1 > \gamma(1+\eta)
\]
where \( 1/(1-\gamma) \) equals the intertemporal elasticity of substitution. The preponderance of empirical evidence suggests that this is relatively small, certainly well below unity, so that we shall restrict \( \gamma < 0 \). The parameter \( \eta \) represents the elasticity of leisure in utility, while \( \theta \) measures the relative importance of public consumption in private utility.

This maximization is subject to the agent’s capital accumulation constraint
\[
\dot{K}_i(t) = [(1-\tau_k)r(t) - \delta]K_i(t) + (1-\tau_w)\omega(t)(1-l_i(t)) - (1+\tau_c)C_i(t) + T_i
\]
where \( \tau_k, \tau_w, \tau_c \) and \( T_i \) are, respectively, the tax rates on capital income, labor income, consumption, and lump-sum transfers that the agent takes as exogenously given, and to the non-negativity constraints on time use, \( 0 < l_i < 1 \) and \( 0 < L_i < 1 \). Our assumption of perfect capital markets implies that poorer individuals may potentially hold negative capital stocks, and this could be the case if such agents anticipate a high future wage. The agent’s optimality conditions are

---

10 As we will see below, richer individuals enjoy more leisure and this could imply that with a sufficiently high wealth endowment an individual chooses not to work. To ensure interior solutions to the allocation of time, we suppose that the initial distribution of wealth is such that all individuals supply a strictly positive amount of labor. Using the individual budget constraint and our results on the steady state stock of capital derived below, it is possible to show that around the steady state \( K > 0 \) if \( k_{i0} < \bar{k} = 1 + \delta(0)(s^*_K(K,L) - \eta Ls^*_K(K,L))/(\eta Ls^*_K(K,L)) \), where \( s^*_K, s^*_L \) denote the equilibrium after-tax shares of capital and labor, respectively, tilde denotes steady-state values, and \( \delta(0) \) is defined in (18) below.


12 Since we are focusing on interior solutions, there is no need to append the inequality constraints on labor/leisure.

13 See the discussion in Turnovsky and García-Peñalosa (2008, p. 1411). Whether this occurs depends also upon the nature of the structural change generating the change in relative wealth.
\[ \eta \frac{C_i}{l_i} = w(K, l) \left( \frac{1 - \tau_w}{1 + \tau_c} \right) \quad (5) \]

\[ (\gamma - 1) \frac{\dot{C}_i}{C_i} + \eta\gamma \frac{\dot{l}_i}{l_i} + \gamma \theta \frac{\dot{G}}{G} - \beta = \frac{\dot{\lambda}_i}{\lambda_i} = \delta - r(K, l)(1 - \tau_k) \quad (6) \]

where \( \lambda_i \) is agent \( i \)'s shadow value of capital, and we have used the fact that \( L = 1 - l \) to express the wage and interest rate, \( w, r \), as functions of average leisure, \( l \), rather than of average employment, \( L \).

Equation (5) equates the marginal rate of substitution between consumption and leisure to the [after-tax] price of leisure, while (6) is the Euler equation modified to include the fact that leisure changes over time. A crucial aspect of the latter equation is that it implies that each agent chooses the same growth rate for the shadow value of capital, irrespective of capital and labor endowment.

As discussed initially by Caselli and Ventura (2000), this specification of preferences will result in the independence of the aggregate equilibrium from distribution.

Using (5) we may write the individual’s accumulation equation, (4), in the form

\[ \frac{K_i}{K_i} = r(K, L)(1 - \tau_k) - \delta + \frac{w(K, L)(1 - \tau_w)}{K_i} \left( 1 - l_i \frac{1 + \eta}{\eta} \right) \frac{T_i}{K_i} \quad (7) \]

Equations (5) and (7) incorporate well-known effects of taxation. From (5) we see that the effective tax rate on labor, \( (\tau_w + \tau_c)/(1 + \tau_c) \), is a crucial determinant of the allocation of time, as in Prescott (2004), while equation (7), in turn, establishes that, for a given labor supply, higher taxes on capital income reduce the rate of accumulation of capital.

### 2.2 Government

We assume that the government sets its expenditure and transfers as fractions of aggregate output, in accordance with \( G = gY(t), T = \tau Y(t) \), so that \( g \) and \( \tau \) become the policy variables along with the tax rates. We also assume that it maintains a balanced budget expressed as

\[ \tau_c r(K, L)K + \tau_w w(K, L)(1 - l) + \tau_c C = G + T = (g + \tau)F(K, L) \quad (8) \]

This means that, if \( \tau_w, \tau_k, \tau_c, \) and \( g \) are fixed, as we shall assume, then along the transitional path,
as economic activity and the tax/expenditure base is changing, the rate of lump-sum transfers must be continuously adjusted to maintain budget balance.\textsuperscript{14} In order to abstract from any direct distribution effects arising from lump-sum transfers (which are arbitrary), we shall assume \(T_i(t)/T(t) = K_i(t)/K(t)\) which ensures that \(\int_0^N T_i dt = (T/K)\int_0^N K_i dt = T\), consistent with the government budget constraint. While this assumption is restrictive in that it does not capture the major redistributive impact that direct transfers have, it has the analytical advantage of allowing us to focus on the distributive impact of distortionary taxes.\textsuperscript{15}

### 2.3 The macroeconomic equilibrium

The key relationship in deriving the aggregate dynamics is equation (6) which implies that each agent chooses the same growth rate for the shadow value of capital, irrespective of capital endowment. Using this equation it can be shown that all agents will also choose the same rate of growth of leisure, implying that average consumption, \(C\), and leisure, \(l\), grow at the same common rates\textsuperscript{16}. The macroeconomic equilibrium can then be described by the pair of dynamic equations:

\[
\frac{dK}{dt} = (1-g)F(K, L) - F_L(K, L)\frac{\left(1-\tau_w\right)}{\left(1+\tau_c\right)} - \delta K \tag{9a}
\]

\[
\frac{dl}{dt} = \frac{F_k(1-\tau_k) - (\beta + \delta) - \left[(1-\gamma)\frac{F_{kl}}{F_k} - \theta \gamma \frac{F_{kl}}{F}ight] \left[(1-g)F(K, L) - F_L(K, L)\frac{\left(1-\tau_w\right)}{\left(1+\tau_c\right)} - \delta K\right]}{\frac{1-\gamma(1+\eta)}{l} - \left[(1-\gamma)\frac{F_{ll}}{F_k} - \theta \gamma \frac{F_{ll}}{F}\right]} \tag{9b}
\]

which determine the joint dynamics of average (aggregate) capital and leisure. The important

\textsuperscript{14} In our simulations we take \(\tau > 0\); \(\tau < 0\) therefore corresponds to a rate of lump-sum taxation. It is important to emphasize that our results on inequality do not depend on the presence of these transfers nor on the form they take. In fact, the same analytical results would be obtained with no transfers. We introduce them in order to focus on the impact of discretionary taxes which we can set and keep fixed during the transition to a steady state, while the transfer rate varies endogenously so as to balance the government’s budget.

\textsuperscript{15} Like all lump-sum transfer schemes, we assume that individual agents are unaware of the rule adopted by the government. An alternative procedure would be to introduce debt financing along the transitional path, but given perfect markets, Ricardian Equivalence implies that this is essentially equivalent to what we are doing. The one difference is that the relative capital stock discussed in Section 3.1 is replaced by relative wealth (capital plus government bonds). But results are essentially identical to those we obtain.

\textsuperscript{16} Taking the time derivative of (5) and combining it with (6) we readily see that \(\dot{C}_i/C_i = \dot{C}/C\) for each \(i\); see Turnovsky and García-Peñalosa (2008) for more details. In particular, \(\rho = \rho_i\) where \(\rho_i\) is constant over time.
observation about this pair of equations is its implication that aggregate behavior is independent of wealth and/or income distributions.

Assuming that the economy is stable, the aggregate dynamic system, (9), converges to a steady state characterized by a constant average capital stock, labor supply, and leisure time, denoted by $\tilde{K}$, $\tilde{L}$ and $\tilde{t}$, respectively. Setting $\dot{K} = \dot{L} = 0$, the steady state is summarized by

\[(1 - \tau_i)F_k(\tilde{K}, \tilde{L}) = \beta + \delta \quad \quad (10a)\]

\[(1 - g)F(\tilde{K}, \tilde{L}) - F_i(\tilde{K}, \tilde{L})\tilde{t}\left(\frac{1 - \tau_i}{1 + \tau_i}\right) - \delta \tilde{K} = 0 \quad \quad (10b)\]

\[\tilde{L} + \tilde{t} = 1 \quad \quad (10c)\]

Note that (10b) implies

\[(1 - g)\tilde{F} - \delta \tilde{F} - \tilde{F}_\alpha \tilde{L}(1 - \tau_i) + \tilde{F}_\tau (1 - \tau_i) \left[1 - \tilde{t} \frac{1 + \eta}{\eta}\right] = 0. \quad \quad (11)\]

If we assume that the share of private consumption expenditure, \[\left[(1 - g) - \delta(\tilde{K}/\tilde{F})\right], \] exceeds the after-tax share of labor income, \[\left(\tilde{F}_\tau \tilde{L}/\tilde{F}\right)(1 - \tau_i), \] then (11) imposes the restriction\[17 \]

\[\tilde{t} > \frac{\eta}{1 + \eta} \quad \quad (12)\]

This inequality yields a lower bound on the steady-state time allocation to leisure that is consistent with a viable equilibrium. As we will see below, this condition plays a critical role in characterizing the dynamics of the wealth distribution.

In order to describe the dynamics of the distribution of capital and income, we first need to obtain the dynamics of the aggregate magnitudes. Linearizing equations (9a) and (9b) around steady state yields the local dynamics for $K$ and $l$,

\[\text{This condition is likely to be met. For example in our simulations (1 - g) - \delta(\tilde{K}/\til{F}) = 0.69 and clearly exceeds (1 - \tau)(\til{F}_\tau \til{L}/\til{F}) = 0.50.}\]

9
\[
\begin{pmatrix}
K \\
l
\end{pmatrix} = 
\begin{pmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{pmatrix}
\begin{pmatrix}
K - \tilde{K} \\
l - \bar{l}
\end{pmatrix}
\] (13)

where \(a_{11}, a_{12}, a_{21}\) and \(a_{22}\) are defined in the Appendix.\(^{18}\) There we show that, under the assumption that \(\theta\) is not too large, \(a_{11}a_{22} - a_{12}a_{21} < 0\), implying that the steady state is a saddle point. The stable paths for \(K\) and \(l\) can be expressed as

\[
K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t}
\] (14a)

\[
l(t) = \bar{l} + \frac{a_{21}}{\mu - a_{22}}(K(t) - \tilde{K}) = \bar{l} + \frac{\mu - a_{11}}{a_{12}}(K(t) - \tilde{K})
\] (14b)

where \(\mu < 0\) is the stable eigenvalue. In the Appendix we establish that \((a_{22} - \mu) > 0\), implying that the slope of the stable arm has the opposite sign to that of \(a_{21}\). The sign of this expression reflects two offsetting influences of capital on the evolution of leisure. On the one hand, a greater capital stock reduces the return to capital and hence to future consumption, thus increasing desired leisure. On the other, greater \(K\) increases wages and thus reduces the growth rate of leisure. Which effect dominates depends upon the underlying parameters, and in particular upon the elasticity of substitution in production, \(\varepsilon\). In the Appendix we demonstrate that for plausible cases [including the conventional case of Cobb-Douglas production and logarithmic utility \((\varepsilon = 1, \gamma = 0)\)] \(a_{21} < 0\), in which case the stable locus is positively sloped; in equilibrium accumulating capital is therefore associated with increasing leisure and \(a_{21}/(\mu - a_{22})\) reflects the strength of this positive association.

For expositional convenience we shall restrict ourselves to what we view as the more plausible case of a positively sloped stable locus, (14b).

The evolution of average leisure will be shown below to be an essential determinant of the dynamics of wealth and income inequality, hence it is important to consider in detail how leisure

\(^{18}\)The reliability of employing linearization to evaluate the dynamic adjustments is examined by Atolia, Chatterjee and Turnovsky (2010), who consider responses to changes in government expenditure in an aggregate one-sector growth model. Their results indicate that linearization yields a satisfactory approximation in a context like this, where the stable manifold is first order, and in addition the transitional dynamics remain close to steady state. As is seen in Section 3.1 below, relative wealth dynamics is then studied by substituting the solution from (13) into (15). However, since the resulting equation is linear in the relative capital stock, \(k\), the quality of the approximation obtained at the aggregate level is maintained in characterizing the distributional dynamics.
reacts to a policy shock. Since \((14b)\) holds at all times, we have
\[
I(0)\bar{I} = \frac{a_{21}}{\mu - a_{22}} (K_0 - \bar{K}). \tag{14b'}
\]

Whenever the economy is subject to a policy shock that results in an increase in the steady-state average per capita capital stock relative to its initial level \((K_0 < \bar{K})\), there will be an initial jump in average leisure, such that \(I(0) < \bar{I}\), so that, thereafter, leisure will increase monotonically during the transition; an analogous relationship applies if \(K_0 > \bar{K}\). We should point out, however, that in our simulations the net impact of the two offsetting influences of the accumulating capital stock on the transition of leisure [as measured by the slope of \((14b)\)] is small. Consequently the response of leisure following the fiscal shock is completed almost immediately, on impact.

3. The distribution of income and wealth

3.1 The dynamics of the relative capital stock

To derive the dynamics of individual \(i\)'s relative capital stock, \(k_i(t) \equiv K_i(t)/K(t)\), we sum \(7\) over the \(i\) agents to obtain an expression for the evolution of \(K(t)\) and combine it with the individual capital dynamics in \(7\). With transfers set such that \(T_i/K_i = T/K\) this leads to\(^{19}\)
\[
\dot{k}_i(t) = \frac{w(K, l)(1 - \tau_w)}{K} \left[ 1 - l \left( 1 + \frac{1}{\eta} \right) - \left( 1 - l \left( 1 + \frac{1}{\eta} \right) \right) k_i(t) \right] \tag{15}
\]
where \(K, l\) evolve in accordance with \((14a, 14b)\) and the initial relative capital \(k_{i,0}\) is given from the initial endowment. First, we note from \((15)\) that agent \(i\)'s steady-state leisure, \(\bar{l}_i\), and share of capital stock, \(\bar{k}_i\), are related by
\[
\tilde{l}_i - \bar{l}_i = \left( l \left( 1 + \frac{\eta}{1 + \eta} \right) \right) \left( \bar{k}_i - 1 \right) \tag{16}\]

This equation captures one of the critical elements determining the evolution of the distributions of

\(^{19}\) For more of the details see Turnovsky and Garcia-Penalosa (2008). We have also considered an alternative lump-sum transfer rule \(T_i = T\), with very small differences in results from those we are reporting here.
wealth and income, and explains why the evolution of the aggregate quantities are unaffected by
distributional aspects. On the one hand, the agent’s labor supply is a linear function of his relative
capital. On the other, the sensitivity of labor supply to relative capital is common to all agents, and
depends upon the aggregate economy-wide leisure. As a result, aggregate labor supply depends only
on the aggregate amount of capital but not on its distribution across agents. Moreover, recalling (12),
equation (16) implies that the greater an agent’s steady-state relative wealth, the more leisure he
consumes and the less labor he supplies. This generates an equalizing effect that partly offsets the
impact of wealth inequality on the distribution of income.

In order to analyze the evolution of the relative capital stock, we linearize equation (15)
around the steady-states \( \tilde{K}, \tilde{L}, \tilde{k}, \tilde{l} \).\(^{20}\) From (12), the coefficient in (15) on \( k_i \) is positive, implying
that in order for agent \( i \)’s relative stock of capital to remain bounded, and therefore yield a finite
steady-state wealth distribution, \( k_i(t) \) must follow the stable path:

\[
k_i(t-1) = \delta(t)(\tilde{k_i} - 1)
\]

where

\[
\delta(t) \equiv 1 + \left( \frac{F_L(\tilde{K}, \tilde{l})(1-\tau_w)}{\tilde{K}} \right) \left( 1 - \frac{l(t)}{\tilde{l}} \right) \left( \frac{F_L(\tilde{K}, \tilde{l})(1-\tau_w)}{\tilde{K}} \right) \left( \frac{\tilde{i} \left( 1 + \frac{\eta}{\mu} \right) - 1}{\eta} - \mu \right)^{-1}
\]

Setting \( t = 0 \) in (17) and (18) yields

\[
\tilde{k_i} - 1 = \frac{1}{\delta(0)} (k_{i,0} - 1).
\]

This is, the steady state relative capital of agent \( i \), which depends both on \( k_{i,0} \), i.e. on the initial
(given) distribution of capital endowments – and on the value at \( t = 0 \) of the coefficient driving the
dynamics of wealth distribution, \( \delta(t) \).\(^{21}\)

\(^{20}\) Note that equations (10) determine \( \tilde{K}, \tilde{L} \). Given these aggregate quantities, the remaining two variables, \( \tilde{k}, \tilde{l} \) are
determined by (16) together with (19) below and depend upon the agent’s initial relative stock of capital.

\(^{21}\) Equation (19) enables us to address whether or not individuals always hold positive capital. From (19) we see that
\( \tilde{k_i} \geq 0 \) if and only if \( k_{i,0} \geq (1-\delta(0))/\delta(0) \). If all agents begin with non-negative holdings of capital (\( k_{i,0} \geq 0 \)) and the
structural change generates an expansion in the aggregate long-run capital stock, (14b) and (18) imply \( \delta(0) > 1 \), so that
this condition is always met. Moreover, since the transition in \( k_i(t) \) is monotonic, \( k_i(t) > 0 \) along the entire transitional
Using (17)–(19), and equations (14), describing the evolution of the aggregate economy, we can express the time path for $k_i(t)$ in the form

$$k_i(t) - \tilde{k}_i = e^{\mu t} (k_{i,0} - \tilde{k}_i), \quad (20)$$

from which we see that $k_i(t)$ also converges to its steady state value at the rate $\mu$.

Note that equations (15) to (18) imply that of all the policy instruments only the tax on labor income has a direct effect on relative capital accumulation. Other instruments will impact $k_i(t)$ only through the changes they induce in aggregate magnitudes, including the rate of adjustment, $\mu$. In particular, government expenditure has no direct effect, despite the fact that it affects the marginal utility of consumption (see equation (5)) and hence affects both savings and hours of work. The reason for this is that, with homothetic preferences, the relationship between the growth rate for the shadow value of capital and for government expenditure is the same for all agents, irrespective of their endowments [see (6)]. As a result it has no direct impact on the relative rate of accumulation and hence on the distribution of wealth.

### 3.2 The dynamics of relative income

An important aspect of income taxes is their direct redistributive effect, causing us to distinguish between before-tax and after-tax income distributions, with the latter measure being arguably of greater significance. We define the relative before-tax and after-tax income of individual $i$ at time $t$ as

$$y_i = \frac{rK_i + w(1-l_i)}{rK + w(1-l)}, \quad (21)$$

and

$$y_i^\tau = \frac{rK_i(1-\tau_i) + w(1-l_i)(1-\tau_\tau)}{rK(1-\tau_i) + w(1-l)(1-\tau_\tau)}, \quad (22)$$

respectively. Note that the after-tax income measure ignores the direct distributional impacts of lump-sum transfers, which are arbitrary.

---

path. For $k_i(t) > 0$ to hold following a contractionary shock, $k_{i,0} \geq \frac{(1-\delta(0))/\delta(0)}{\tilde{k}}$ imposes a positive lower bound, on each agent’s initial holding of capital.
The relative before- and after-tax income of agent \( i \) may then be expressed as

\[
y_i(t) - 1 = \varphi(t)(k_i(t) - 1),
\]

\[
y^a_i(t) - 1 = \psi(t)(k_i(t) - 1)
\]

where

\[
\varphi(t) \equiv s(t) - (1 - s(t)) \frac{l(t)}{1 - l(t)} \left(1 - \frac{1}{1 + \eta}\right) \frac{1}{\delta(t)},
\]

\[
\psi(t) \equiv \varphi(t) + \frac{s(t)(\tau_w - \tau_k)}{s(t)(1 - \tau_k) + (1 - s(t))(1 - \tau_w)}(1 - \varphi(t))
\]

Consider, first the expression for before-tax income inequality, \( \varphi(t) \). This measure has two components, the share of capital income, \( s(t) \), and relative labor income, reflected in the second term of (25) and which captures the fact that less wealthy agents supply more labor. In the case of after-tax relative income, we see that both income tax rates, \( \tau_k \) and \( \tau_w \), exert two effects on the after-tax income distribution. First, by influencing \( \varphi(t) \), they influence gross factor returns, and therefore the before-tax distribution of income. But, in addition, they have direct redistributive effects that are captured by the second term on the right hand side of (26). Post-tax income inequality will be less than pre-tax income inequality if and only if \( \tau_w < \tau_k \). In contrast, the redistributive effects of a consumption tax on post-tax inequality are only indirect and operate to the extent that \( \tau_k \neq \tau_w \).

### 3.3 The distributions of wealth and income

We can now compute indexes of inequality. Because of the linearity of (17), (23) and (24) we can immediately transform these equations into corresponding relationships for the standard deviations of the distributions of capital and income across agents, which therefore serve as convenient (and tractable) measures of wealth and income inequality. From (17) we can express the standard deviation of the distribution of relative capital at time \( t \) as
\[ \sigma_k(t) = \delta(t) \sigma_k = \frac{\delta(t)}{\delta(0)} \sigma_{k,0}. \quad (17') \]

Equation (20) allows us to write \( \sigma_k(t) - \tilde{\sigma}_k = e^{\rho t} (\sigma_{k,0} - \tilde{\sigma}_k) \), from which we see that the degree of wealth inequality does indeed vary during the transition, converging to the long-run distribution \( \tilde{\sigma}_k \). \(^{22}\) In addition, \((17')\) together with \((18)\) and \((14b')\) imply that

\[ \sigma_{k,0} > \sigma_k(t) > \tilde{\sigma}_k \quad \text{if and only if} \quad K_0 < K(t) < \tilde{K}. \quad (27) \]

Thus we conclude that if the economy undergoes an expansion (contraction) in its capital stock then wealth inequality will decrease (increase) during the transition, and the long-run distribution of wealth will be less (more) unequal than is the initial distribution.

Further intuition for this result can be easily seen by noting from \((17)\) - \((19)\), that

\[ \text{sgn}(\tilde{k}_l - k_{l,0}) = \text{sgn}(\tilde{k}_l - 1)(l(0) - \bar{l}) \]

Recall that if the economy converges to the steady state from below, then \( l(0) < \bar{l} \). For people who end up above the mean level of wealth, their relative wealth will have decreased during the transition \( \tilde{k}_l < k_{l,0} \), while for people who end up below the mean level of wealth, their relative wealth will have increased, \( \tilde{k}_l > k_{l,0} \), implying a narrowing of the wealth distribution. Equations \((18)\) and \((19)\) further imply that the closer \( l(0) \) jumps to its steady state, \( \bar{l} \), the smaller the subsequent adjustment in \( l(t) \), and hence the smaller is the overall change in the wealth distribution. This is because if the economy and therefore all individuals fully adjust their respective leisure times instantaneously, they will all accumulate wealth at the same rate, causing the wealth distribution to remain unchanged.

Similarly, because of the linearity of \((23)\) and \((24)\) in \( k_i(t) \), we can express the relationship between before-tax and after-tax relative income and relative capital (wealth) in terms of corresponding standard deviations of their respective distributions, namely

\[ \sigma_y(t) = \phi(t) \sigma_k(t) \quad (23') \]

\[ \sigma^{a_y}(t) = \psi(t) \sigma_k(t) \quad (24') \]

\(^{22}\) However (20) implies that the ranking across agents according to wealth remains unchanged throughout the transition. To obtain changes in these rankings requires an additional source of heterogeneity, such as differential abilities.
If labor is supplied inelastically, \( \varphi(t) = s(t) \) so that \( \sigma_y(t)/\sigma_k(t) = s(t) \). However, when labor is flexible, poor agents supply more labor than do the wealthy, which partially offsets the effect of the unequal distribution of capital. From inequality (12) the term in square brackets in equation (25) is positive and hence \( \varphi(t) < s(t) \), implying that the ratio of wealth inequality to income inequality is less than the share of capital.

Finally, letting \( t \to \infty \), we can express the steady-state distributions of income as

\[
\tilde{\sigma}_y = |\tilde{\varphi}|\tilde{\sigma}_k
\]

(23’’)

\[
\tilde{\sigma}_y^a = |\tilde{\psi}|\tilde{\sigma}_k,
\]

(24’’)

where

\[
\tilde{\varphi} = \lim_{t \to \infty} \varphi(t) = 1 - \frac{1 - \tilde{s}}{1 + \eta} \frac{1}{1 - \tilde{s}} = 1 - \frac{1}{1 + \eta} \frac{F_L(\tilde{K}, \tilde{L})}{F(K, L)},
\]

(25’)

\[
\tilde{\psi} = \lim_{t \to \infty} \psi(t) = \tilde{\varphi} + \frac{s(\tau_w - \tau_k)}{s(1 - \tau_k) + (1 - \tilde{s})(1 - \tau_w)} (1 - \tilde{\varphi}).
\]

(26’)

From (12) and (25’) we see that if

\[
\frac{\eta + \tilde{s}}{1 + \eta} > \tilde{\iota} > \frac{\eta}{1 + \eta}
\]

(12’)

these expressions imply that \( \text{sgn}(\tilde{y}_i - 1) = \text{sgn}(y_{i,0} - 1) \). In that case, the distribution of income converges to a long-run distribution such that the relative ranking of agents according to income is the same as that of capital, as well as that of the initial income distribution. However, if

\[
\tilde{\iota} > \frac{\eta + \tilde{s}}{1 + \eta} \frac{\eta}{1 + \eta},
\]

(12”)

in the long run, agents having above average wealth will have below-average income. This will occur if the tax on labor income is sufficiently high so that individuals are induced to devote a very small fraction of their time to labor. A plausible example of this is given in Section 5.4 below.

Note that the dynamics of the distribution of income differ from those of the distribution of capital in that income inequality (both before-tax and after-tax) jumps in response to a shock. To see
that we rewrite (23') as

$$\sigma_y(t) = s(t)\sigma_y(t) - \left[1 - s(t)\right] \frac{l(t)}{1 - l(t)} \left[1 - \frac{1}{1 + \eta} \right] \tilde{\sigma}_k$$  \hspace{1cm} (28)$$

which indicates that although the distribution of wealth, $\sigma_k(t)$ evolves gradually, the initial jump in leisure, $l(0)$, means that income distribution is subject to an initial jump following a structural or policy shock. Thereafter, it evolves continuously in response to the evolution of the distribution of capital and factor returns.

To assess the long-run effect of policy on income distribution, note that

$$\frac{\tilde{\sigma}_y}{\sigma_{y,0}} = \frac{\tilde{\sigma}_k}{\phi_0 \sigma_{k,0}} \quad \text{and} \quad \frac{\tilde{\sigma}^a_y}{\sigma_{y,0}} = \frac{\tilde{\sigma}_k}{\psi_0 \sigma_{k,0}}$$

where $\sigma_{y,0}$ and $\sigma_{y,0}^a$ denote the initial distributions of before and after-tax income. Whether the long-run distribution is more or less unequal than the initial distribution depends on the long-run change in the distribution of capital, as reflected in $\tilde{\sigma}_k/\sigma_{k,0}$, factor returns, as reflected in $\tilde{\phi}/\phi_0$, and—in the case of the after-tax distribution—the size of the wage tax relative to the capital tax rate.

### 3.4 Distribution of welfare

Recalling (3), agent $i$’s welfare at time $t$ is $\Omega_i(t) \equiv \left(1 + \frac{1}{\gamma}\right)C_i(t)\gamma l_i(t)^{\eta} G^{\rho_i}$. Using equation (5) and the fact that $l_i(t) = \rho_i l(t)$ [see footnote 16], implies:

$$\frac{\Omega_i(t)}{\Omega(t)} = \left(\rho_i\right)^{(1+\gamma)}$$  \hspace{1cm} (29)$$

where $\Omega(t)$ is the average welfare level at time $t$, and from (16)

$$\rho_i = 1 + \left(1 - \frac{\eta}{(1 + \eta)l} \right) (\tilde{k}_i - 1)$$

Substituting (29) into (3), yields an analogous relationship for the relative intertemporal welfare, $W_i$, evaluated along the equilibrium growth path:
\[ \frac{W_i}{W} = \frac{\Omega(i)}{\Omega(t)} = (\rho)^{\eta(1+\eta)} \]  

At each instant of time, agent i’s relative welfare remains constant, so that his intertemporal relative welfare, \( \frac{W_i}{W} \), remains constant as well.

We can now compute a measure of welfare inequality. A natural metric for this is obtained by applying the following monotonic transformation of relative utility, enabling us to express the relative utility of individual \( i \) as

\[ \left( \frac{W_i}{W} \right)^{\eta(1+\eta)} = \left( \frac{\Omega(i)}{\Omega(t)} \right)^{\eta(1+\eta)} = 1 + \left( 1 - \frac{\eta}{(1+\eta)^t} \right) (\tilde{k}_t - 1) \]

Both instantaneous and intertemporal welfare inequality, expressed in terms of equivalent units of wealth, can then be measured by the standard deviation of relative utility,

\[ \tilde{\sigma}_u = \left( 1 - \frac{\eta}{(1+\eta)^t} \right) \tilde{\sigma}_k = \tilde{\chi} \tilde{\sigma}_k \]

This expression is identical to that obtained in our earlier work, García-Peñalosa and Turnovsky (2007). Note that, in contrast to the dynamics associated with income inequality, welfare inequality remains constant during the transition, reflecting the fact that relative leisure is constant.

4. Steady state effects of fiscal policy

Our objective is to examine the effect of different fiscal policies on the time paths of the distributions of wealth and income. We begin by considering the steady-state effects, since with forward-looking agents, these long-run responses are critical for determining the transitional dynamics. In all cases we assume that the economy starts from an initial equilibrium in which government expenditure is fully financed by lump-sum transfers, so that \( g_0 + \tau_0 = 0 \) in equation (8), and initial distortionary tax rates are all zero, i.e. \( \tau_c = \tau_w = \tau_k = 0 \).

In Table 1 we summarize and compare alternative modes of financing a specified increase in government spending. The change in tax rates required by the increase in government expenditure is given by the government budget constraint (8), which we can write in the form
We consider first the effects of an increase in government expenditure, which are obtained when the increase in $g$ is financed by a reduction in transfers. Starting from $\tau_c = \tau_w = \tau_k = 0$ the required reduction in transfers is $d\tau = -dg$ and the long-run responses are reported in the first column. Since distribution is determined by aggregate behavior, we begin with the latter, which are summarized in the first four rows. These results are standard and require little discussion. The increase in government spending raises the long-run capital stock, labor supply, and therefore output, proportionately, leaving factor shares unaffected.

The distributional impacts are reported in Rows 5-8. Noting the response of capital (Row 1), in conjunction with (27), we immediately conclude that an increase in government expenditure reduces long-run wealth inequality, $\bar{\sigma}_k$. As discussed in Section 3, pre-tax income inequality is given by $\sigma_y = \bar{\phi}\bar{\sigma}_k$, the response of which depends upon that of both $\bar{\sigma}_k$ and $\bar{\phi}$, reported in Row 6. As already noted, during the transition to the new steady state the distribution of wealth will become less dispersed, tending to reduce income inequality. At the same time, recalling that

$$\bar{\phi} = 1 - \frac{1}{1+\eta} \frac{1-\bar{s}}{1-L}$$

we see that with factor shares, $\bar{s}$, remaining unchanged (Row 4) and with the expansion in government spending increasing labor supply (reducing $\bar{L}$), this causes an offsetting increase in $\bar{\phi}$. To understand the role of the labor supply in this adjustment, we rewrite (16’) in the form

$$\bar{L} = \bar{L} - \left(\bar{I} - \frac{\eta}{1+\eta}\right)\left(\bar{k}_r - 1\right)$$

which implies that individual labor supplies are negatively correlated with capital endowments and have a standard deviation of $\sigma_L = (\bar{I} - \eta/(1+\eta))\bar{\sigma}_L$ which is also the dispersion of relative labor incomes. Thus an increase in labor supply, that is, a decline in $\bar{L}$, implies less dispersion of labor incomes which, since they are negatively correlated with capital endowments, tends to increase income inequality.

Concerning after-tax income inequality, $\sigma_y^a$, we see from (26) that $g$ does not have direct
redistributive effects, implying that its impact on $\psi$ operates entirely through $\varphi$. That is, the effect on pre-tax and post-tax income distributions is the same. The final row reports the response of welfare inequality, which applies uniformly throughout the transition and not just in steady state. The key point is that the adjustment in labor supply now reinforces the effect due to wealth inequality. For example, the fact that an increase in government expenditure reduces aggregate leisure, thereby reducing the dispersions of both labor supply and leisure, implies an overall unambiguous decline in welfare inequality.

In the case of distortionary tax financing, we assume that the corresponding tax rate is set such that it fully finances the long-run change in government expenditure. Thus, again starting from $\tau_c = \tau_w = \tau_k = 0$, the corresponding required changes in the three tax rates are respectively

$$
\frac{d\tau_c}{dg} = \left(\frac{\bar{F}}{\bar{C}}\right)dg, \quad \frac{d\tau_w}{dg} = (1/(1-s))dg, \quad \frac{d\tau_k}{dg} = (1/s)dg
$$

This means that during the transition residual financing using lump-sum tax financing must also be employed to ensure that the budget remains balanced at all times. Thus in general, the response of variable $X$, say, to a distortionary tax-financed increase in government expenditure is given by

$$ \left. \frac{dX}{dg} \right|_{\tau} = \frac{\partial X}{\partial \tau_c} \frac{\partial \tau_c}{\partial g} + \frac{\partial X}{\partial \tau_w} \frac{\partial \tau_w}{\partial g} + \frac{\partial X}{\partial \tau_k} \frac{\partial \tau_k}{\partial g} \quad \tau_i = \tau_c, \tau_w, \tau_k $$

In the case where the increase in government expenditure is financed using a consumption tax, we see from column 2 in Table 1 that the net effects are zero. It is straightforward to show that the impact of an increase in the consumption tax on all aggregate magnitudes are opposite to those of government expenditure, and can be explained by the same intuition. On balance this mode of financing has no impact on either aggregate quantities or on distribution. This is unsurprising. First, the reduction in consumption due to the higher tax is exactly offset by the higher marginal utility of consumption induced by the increase in $g$, thus leaving capital and labor unchanged. Second, the consumption tax has no direct redistributive implications. As a result neither aggregate magnitudes nor distributional variables change.

Aggregate effects are more complex in the case of taxes on capital and labor since they combine the expansionary effect of increased expenditure with the contractionary effect of higher
distortionary taxation. When the expansion is financed by a tax on wage income the distortionary tax effect dominates the direct expenditure effect. As a result aggregate capital, labor, and output all fall proportionately. An increase in \( g \) financed by the tax on capital income raises the long-run marginal physical product of capital, implying that it reduces the capital-labor ratio. Although capital declines unambiguously, labor will do so only if the elasticity of substitution in production, \( \varepsilon \), is sufficiently large.\(^{23}\) While output also declines unambiguously, the net effect on the factor shares depends upon \((\varepsilon - 1)\).\(^ {24}\)

In contrast to financing by reducing transfers, an increase in either \( \tau_w \) or \( \tau_k \), by decreasing the long-run capital stock, results in an increase in wealth inequality. With a wage income tax, \( \phi \) also declines thus offsetting the wealth effect. Recall that a higher tax on labor makes the fiscal system less progressive, the reason being that labor income is less unequally distributed than is capital income. As a result the impact of the policy on \( \bar{\phi} \) combines the direct effect that tends to reduce post-tax inequality (captured by the term \( \bar{s}(1-\bar{\phi})/(1-\bar{s}) \)) and the indirect on stemming from the change in \( \bar{\phi} \), so that post-tax inequality may increase or decrease. In contrast, welfare inequality is unambiguously lower as the effects on income distributions combine with a more dispersed labor supply.

The impact of \( \tau_k \) on \( \bar{\phi} \) is more complex since it involves three effects. First, during the transition to the new steady state with a lower stock of capital the distribution of wealth will become more dispersed. Second, \( \bar{\phi} \) falls or increases depending on whether as the new steady state generates a lower \( \bar{L} \). Third, the reduction in the capital-labor ratio implies a change in the capital share, \( \bar{s} \), the impact of which depends on whether \( \varepsilon < 1 \). In the case of the Cobb-Douglas production function this effect is zero and since in this case \( \partial L/\partial \tau_k < 0 \), then \( \phi \) and also \( \phi \) falls. For the Cobb-Douglas, the direct redistributive effect of the capital income tax reinforces the pre-tax income inequality effect and post-tax income inequality falls relative to wealth inequality.

\(^{23}\) For example, \( \varepsilon > \bar{s} \) is a weak condition to ensure that employment declines.
\(^{24}\) This is not surprising since \( \tau_k \) is equivalent to a decline in productivity, the effect of which on factor shares is well known to depend upon \((\varepsilon - 1)\).
5. Numerical Simulations

To obtain further insights into the dynamics of wealth and income distribution we employ numerical simulations. These are based on the following functional form and parameter values, characterizing the benchmark economy:

<table>
<thead>
<tr>
<th>Production function: ( Y = AK^{\alpha}L^{1-\alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic parameters: ( A = 1.5, \alpha = 0.36 )</td>
</tr>
<tr>
<td>( \beta = 0.04, \gamma = -1.5, \eta = 1.5, \theta = 0.3, \delta = 0.06 )</td>
</tr>
<tr>
<td>Fiscal parameters: ( \tau_c = 0.276, \tau_w = 0.224, \tau_k = 0.08, g = 0.15 )</td>
</tr>
</tbody>
</table>

Preferences remain specified by the constant elasticity utility function, (3), with the intertemporal elasticity of substitution \( 1/(1-\gamma) = 0.4 \), rate of time preference of 4%, while the elasticity of leisure in utility is 1.5, and that with respect to government expenditure 0.3. The production function is of the Cobb-Douglas form, with distributional parameter \( \alpha = 0.64 \), and \( A = 1.5 \) scales the level of productivity. The depreciation rate is 6% per annum. These parameters are all very standard and typical of those found in the literature.

The choice of tax rates is less straightforward and has generated debate, due to the difficulty of mapping the complexities of the real world tax structure into a simple one-sector growth model. We use the effective tax rates on consumption, labor and capital constructed by McDaniel (2007), following the methodology proposed by Mendoza et al. (1994). The rates listed above are the US averages for the decade 1991-2000. A key feature is that for the US economy \( \tau_k > \tau_w \), a characteristic that holds uniformly since 1953. Finally, setting government consumption expenditure rate at \( g = 0.15 \) approximates the US experience in the 1990’s.

Our measures of inequality are reported in terms of the standard deviation measures employed in our theoretical discussion, with the initial standard deviation of relative capital (wealth)

---

25 For simplicity we assume that depreciation costs are not tax deductible. In the event that they are \( \delta = 0.06 \) is equivalent to a pre-tax depreciation rate of around 8.3%.

26 For example, the intertemporal substitution of substitution of 0.4 is well within the range summarized by Guvenen (2006), while the relative weight on leisure in utility is close to the conventional value of the real business cycle literature; see Cooley (1995). The production elasticity \( \alpha = 0.36 \) dates back to at least Kydland and Prescott (1982) and has been adopted consistently since; see e.g. Papageorgiou and Pérez-Sebastian (2007) for a recent example.

27 This is documented by Turnovsky (2004).
being normalized to 1. We suppose that the economy is initially in steady state, and consider the impact of policy changes on both the new steady-state distributions and their transitional dynamics.

Line 1 of Table 3 reports the benchmark equilibrium for the chosen benchmark parameters. There we see that $\gamma = -1.5$ together with $\eta = 1.5$ yields an equilibrium allocation of labor of 30.7% and an implied Frisch elasticity of labor supply of around 1.2, both of which are consistent with existing empirical evidence. The elasticity $\theta = 0.3$ implies that the optimal ratio of public to private consumption is 0.3, which is reasonably close to the observed ratio of 0.28 for the US during the 1990’s. The depreciation rate in conjunction with $\alpha$ is the critical determinant of the rate of convergence. Setting $\delta = 6\%$ yields a convergence rate of around 5.6%, very close to the value of 5.8% suggested by Evans (1997) as being a more accurate estimate of the true value. The implied share of output devoted to transfers is 14.8%, which implies that total tax revenue is 29.8%, very close to that observed in the US during the period 1995-2000.

One of the issues confronting policymakers in both the United States and Europe is the problem of maintaining government spending, given the current level of debt. Without introducing government debt explicitly, which a detailed analysis of this issue would require, our framework can nevertheless provide some preliminary insight into this question and the constraints that current debt levels impose. More specifically, the exercises that we have been describing formally, and that we analyze numerically, can be viewed as studying how to finance government spending and transfers, while holding the long-run government debt to GDP ratio constant.

---

28 The Frisch elasticity of labor supply describes the elasticity of hours worked with respect to the wage rate, given a constant marginal utility of wealth. For the utility function (3) it is $(dL/L)/(d(w/L)) = ([1/(1-\gamma)](1-\gamma))/((1-\gamma/(1+\eta)))).$ Empirical estimates vary extensively across gender and race; see Browning, Hansen, and Heckman (1999). Our implied value 1.2 is close to that (1.40) obtained in the early study of Lucas and Rapping (1970). It is also consistent with more recent estimates (around 1) obtained by Chang and Kim (2006) and Kimball and Shapiro (2008).

29 Original benchmark estimates of the convergence rate of around 2% were established by Barro (1991), and Mankiw, Romer and Weil (1992), among others. Subsequent studies have suggested that these estimates may be substantially downwardly biased; see e.g. Islam (1995), Caselli, Esquivel, and Lefort (1996), and Evans (1997).

30 The implied transfer share is higher than what we tend to observe in the data. The reason for this is that we are abstracting from public investment. The distributional impact of government investment expenditures is a complex question which is beyond the scope of this paper; see for example Calderon and Servén (2004).

31 To see this, assume that government bonds, $B$, say, are perfect substitutes for capital and are taxed at the same rate. Recalling (10a), the steady-state government budget constraint (8’) becomes $\tau_s + \tau_c(1-s) + \tau c/F = g + \tau + \beta [B/F].$
5.1. Financing an increase in government expenditure

Table 2 reports the effect of an increase in government expenditure from its base rate of 15% of output by 5 percentage points to 20% of output. To consider the distributional implications of the expenditure increase, we define the initial steady-state distributions of pre-tax income and post-tax income (prior to any shock) by the respective quantities:

\[
\tilde{\sigma}_{y,0} \equiv \varphi_0 \sigma_{k,0} = \left[1 - \frac{1 - s(\tilde{K}_{0}, \tilde{L}_{0})}{1 - l_0} \right] \sigma_{k,0},
\]

\[
\tilde{\sigma}_{y,0}^a \equiv \psi_0 \sigma_{k,0} = 1 - \left(\frac{1 - \tilde{\sigma}_{y,0}}{\sigma_{k,0}}\right) \frac{1 - \tau_w}{1 - \tau_k + (1 - \delta)(1 - \tau_k)}
\]

where we normalize the initial given distribution of capital by \(\sigma_{k,0} = 1\). Since \(\tau_k > \tau_w\) in the benchmark economy, the fiscal system entails direct redistribution and the initial equilibrium implies standard deviations of pre-tax and post-tax income distributions of \(\tilde{\sigma}_{y,0} = 0.165 > \tilde{\sigma}_{y,0}^a = 0.144\), both of which overstate the degree of welfare inequality, \(\sigma_u = 0.135\).\(^{32}\)

The next three rows report the numerical effects of financing an (arbitrary) increase in government expenditure of 5 percentage points by a reduction in transfers (i.e. a lump-sum tax), an increase in the wage income tax, and an increase in the capital income tax, respectively. Lump-sum tax financing entails an increase in output, as government expenditure encourages private consumption, leading to an increase in labor supply and capital stock, while the distortions created by capital or wage income taxes result in reduced output and less capital. These results are qualitatively identical to the theoretical results reported in Table 2.

The effects on distribution of the three modes of financing contrast sharply, in line with Table 2. In all cases, the long-run changes in wealth inequality are small, reflecting the fact that most of the adjustment in \(L\) occurs instantaneously. As a result, the changes in income distribution are dominated by the relative income effects, \(\tilde{\varphi}\) and \(\tilde{\psi}\). Thus, while lump-sum taxation results in a

\(^{32}\) The final column reports the standard deviation of labor supply. Information on this is sparse. The only estimate we have been able to obtain is one of around 0.361 using German data for 1995-1996 obtained by Heer and Maussner (2009, p.381), to which our solution is quite close. But given the role of labor supply it would seem that the implications for the dispersion of labor supply across agents merits more detailed empirical investigation.
small reduction in wealth inequality it leads to substantial increases in pre-tax and post-tax income inequality of around 25% and 29%, respectively. These effects on inequality are particularly large as, although the lump-sum tax has (by construction) no direct distributive effect, the increase in government expenditure results in a sharp rise in the labor supply, which in turn makes the distribution of income more dispersed. Concerning the effect of financing by a tax on labor, since the contractionary effect of the tax dominates the expenditure effect, this results in a lower labor supply which reduces pre-tax inequality, by over 14%. At the same time the redistributive effect makes the tax structure more regressive. This effect dominates, and post-tax income inequality rises. The opposite occurs with a capital income tax. The increase in labor supply, together with the more than proportionate reduction in the capital stock, reduce wages, leading to a more dispersed distribution of pre-tax income. This is nevertheless dominated by the increased progressivity of the tax system, resulting from the redistribution, leading in a reduction in after-tax inequality by 29.5%. Although these figures are large, they are of the order of magnitude of observed cross-country differentials. For example, in 2000 the Gini coefficient of disposable income was 28% in France and Germany and 37% in the US, implying that the latter was 34 percent higher than the former.\footnote{These Gini coefficients are those reported by the Luxembourg Income Study key figures for equivalized household disposable income (see http://www.lisproject.org/).}

Fig. 1 plots the time paths of the economy in response to the policy changes. The left-side panels illustrate the aggregate magnitudes, capital, labor and income, while the right-side panels represent the distributions of wealth, pre-tax and post-tax income. Both for the aggregate quantities and the distribution measures, the quantities are measured relative to their initial pre-shock, steady-state values.\footnote{The paths corresponding to the different quantities are as identified in the third panel of the figure.} Note that while the aggregate capital stock and its distribution evolve continuously from their initial values, labor supply, output, and both measures of income distribution undergo initial jumps, arising from the initial jump in leisure. In fact, the bulk of the adjustment in labor supply occurs through its initial jump, leading to little change in wealth inequality, although it is more pronounced in the case of capital income tax financing.

As an example, consider the impact of an increase in $g$ financed by a wage income tax. Three effects are in operation. The first is due to the fact that the economy converges to a smaller capital
stock, leading to an increase in wealth inequality, although as just noted this effect is small. The second is the effect on income distribution through the change in work hours. The reduction in the labor supply implies that the distribution of labor incomes becomes more dispersed, and since there is negative correlation between individual capital and labor supply, this tends to reduce income inequality. Because of the initial jump in leisure, this comes into effect immediately and in our simulations is always sufficiently strong to offset the impact of a more unequal distribution of capital and pre-tax income inequality increases. Lastly, because wage income is more equally distributed than is capital income, the direct effect of the tax is to increase post-tax inequality. Since the increase in the tax rate is large (\( \tau_w \) goes from 22.4% to 30.8%) this effect is strong and results in a more unequal distribution of post-tax income than before the policy change.

Three further aspects stand out from this figure. The first is the sharply contrasting effects of \( \tau_w \) and \( \tau_c \) on the pre-tax and post-tax income distributions, as well as on welfare inequality. Financing an increase in \( g \) by a wage tax results in a more equal distribution of pre-tax income, but more unequal distributions of post-tax income and welfare. The opposite holds when the capital income tax is used. The second concerns the tradeoffs that exist between increases in output and reductions in inequality. Whereas lump-sum tax financing of the increase in expenditure raises long-run output by over 5% it increases both inequality measures by well over 20%, while welfare inequality actually declines by over 15%. In contrast, capital income tax financing reduces post-tax inequality by 29.5%, and welfare inequality slightly by 2.22%, but reduces output by 9.9%. Wage income tax financing is a rather unattractive policy, as it reduces output by 2.8% and raises post-tax income inequality by over 7% and welfare inequality by over 8%. Third, the primary response of income inequality occurs instantaneously, with relatively little adjustment over time. This reflects (i) the corresponding time path for labor supply, as a result of which there is little change in wealth inequality, and (ii) the Cobb-Douglas production function, consistent with (28). The smaller initial jump in labor in the case of capital income tax financing implies that inequality is subject to more transitional dynamics. Nevertheless, the response of the distribution of wealth is small compared to the effect of changes in both labor supplies and factor prices.
5.2 Distributional effects of higher taxes

Consider now an increase in transfers of 5 percentage points financed by increasing either the wage income tax, capital income tax, or consumption tax. Our specification of the transfers renders them distributionally neutral, and the only effect will be due to the impact of the higher distortionary tax rates. Starting from the same initial steady state as in the previous exercise, Fig. 2 illustrates the dynamics of both the economy-wide aggregates and the alternative distributional measures.

All three forms of distortionary taxation result in a decline in capital stock, labor supply and average output. Hence, in all three cases wealth inequality increases (slightly), while both pre-tax and post-tax income inequality decline. As discussed earlier, a reduced labor supply implies more dispersion of labor incomes and, since they are negatively correlated with capital endowments, tends to reduce income inequality. The wage income tax generates the largest reduction in pre-tax inequality, but the smallest in terms of post-tax inequality due to the direct effect of the tax. The consumption tax is clearly preferable to the wage tax, since it entails a smaller output loss (4.4% instead of 7.2%) while the reduction of after-tax income inequality is nearly 40% larger than that entailed by a higher wage tax. Increasing the capital income tax yields the largest reduction in post-tax income inequality, partly due to the fact that the tax rate increases from 27.6% to over 40%. However, this reduction in income inequality is obtained at a substantial cost in terms of output (it falls by 13.2%) and an 11.6% increase in welfare inequality. Taken together, Figs. 1 and 2 highlight how the consequences of tax increases on income distribution depend critically upon how the resulting revenues are spent.

5.3 Elasticity of labor supply

The elasticity of labor supply is a key parameter determining distributional responses, both because it impacts the change in the aggregate labor supply and hence factor prices, but also as a key determinant of the relationship between wealth inequality and income inequality [see equation (28)]. Accordingly, we have re-examined the impact of an increase in government spending from 15% to 20%, for alternative values of the elasticity of leisure in the utility function, namely $\eta = 1$ (low) and
As is well-established, a higher elasticity results in a stronger response of the labor supply to tax changes. Our numerical results indicate that it also results in stronger distributional responses, with changes in inequality increasing with $\eta$. For example, if the increase in government spending is financed by a capital tax, the labor supply increases by 1.50% with $\eta = 1$ and by 2.01% with $\eta = 2$, compared to 1.63% in our benchmark case. The response of the distribution of wealth is the same in the three cases (due to the limited transitional labor supply response), but income inequality responses are affected: post-tax inequality falls by 22.6% when $\eta = 1$ and by 34% for $\eta = 2$.

Inspecting equation (24) we can see that a key term driving the inequality response is $\left((1 - L) - \eta / (1 + \eta)\right)/L$. Hence a larger $\eta$ has both a direct negative effect due to the fact that for a given average labor supply incomes are more dispersed, and an indirect one caused by the stronger labor supply response that impacts factor prices. In other words, the key parameter driving the aggregate labor supply response—the elasticity of utility with respect to leisure—is also a crucial determinant of the distributional response.36

5.4. A ‘European’ Economy

Recently, an extensive literature has examined the role of the changing tax structure in Europe relative to the US over the latter part of the 20th century in accounting for differences in labor supply; see e.g. Prescott (2004), Alesina et al (2005), Rogerson (2006) and Ohanian, Raffo and Rogerson (2008). This literature has, however, paid little attention to the distributional implications of these tax changes. This is a very complex issue, involving country-specific institutional aspects. It is made all the more difficult due to the fact that the tax changes as documented by McDaniel (2007) occurred (and are continuing) gradually and that the time profile of structural changes has itself potentially important distributional consequences; see Atolia, Chatterjee, and Turnovsky.

35 We also examined different elasticities of substitution, namely 0.75 and 1.25 and obtained results that were qualitatively the same as in our benchmark case. Robustness analysis using different values for the elasticity of utility with respect to government expenditure indicate that although the speed of convergence depends on this parameter, its impact is small and has virtually no effect on the dynamics or steady state of distribution.

36 The qualitative properties of the dynamics are very similar for the benchmark case and are not illustrated.
(2010). Thus, although space limitations preclude any detailed analysis of this important issue, nevertheless McDaniel’s data highlights crucial structural differences between US and the main European countries, enabling some initial observations to be made. These arise from the fact that whereas the US has a relatively high tax on capital and lower taxes on labor and consumption, these relative rates are reversed in Europe.

Table 4 summarizes the long-run effects of a 5 percentage point increase in government consumption for a European economy. The chosen base tax rates $\tau_k = 0.17, \tau_c = 0.21, \tau_w = 0.36$ are averages for France and Germany over the decade 1991-2000, obtained from McDaniel (2007), while the base rate $g = 0.20$ reflects the fact that government consumption is around 5 percentage point higher in Europe than in the US.

The base line in Table 4 is consistent with the documented fact that the higher tax on labor income in Europe implies a substantial reduction in working hours. As for the US, the initial distribution on capital is normalized at $\sigma_k = 1$, which means that we can make only relative comparisons of distribution across the two economies, rather than absolute comparisons. The fact that $\tau_w > \tau_k$ now implies that the ranking between income inequality and welfare inequality changes dramatically. Whereas for the US $\sigma_y > \sigma_{y,w} > \sigma_u$, for Europe these rankings are reversed, $\sigma_u > \sigma_{y,a} > \sigma_y$. Moreover, European pre-tax income inequality is very small. The reason for this can be seen from (25'), where the high tax on labor income reduces $\tilde{L}$, thus reducing $\tilde{\phi}$.

In Europe, lump-sum tax financing of government spending is more expansionary, capital income tax financing is less contractionary, and wage income tax financing is more contractionary, as compared to the US. This reflects the increasing inefficiency associated with increasing an already high tax rate. The qualitative distributional effects of the alternative modes of financing government expenditure are the same as in the US. In percentages, the magnitudes on the income inequality measures are dramatically different, due to the benchmark inequality being so small (0.0399). One interesting aspect is that financing with a wage income tax implies that (12") applies so that the reduction in pre-tax income inequality is associated with a reversal of agents’ relative income and wealth position.
6. Concluding comments

The neoclassical growth model is one of the cornerstones of modern macropolicy analysis. Although the model has been extensively used to assess the impact of different forms of taxation on aggregate magnitudes, the implications of changes in tax rates on the distribution of income and wealth has received only limited attention. In this paper we have examined the distributional responses to tax changes in such a model in which agents differ in their initial endowment of capital.

The endogeneity of the labor supply plays a crucial role in our analysis. The key mechanism whereby the initial distribution of capital endowments influences the distribution of income is through the wealth effect, which implies that wealthier agents supply less labor, although the resulting distribution of labor supplies is less unequal than that of the capital endowments. Ceteris paribus tax changes then have three distributional effects. First, during the transition to the new steady state, agents will accumulate capital at different rates, implying a change in the distribution of wealth. Second, higher taxes reduce the labor supply of all agents, but decrease that of the capital-rich by more than that of the capital-poor. This results in a more dispersed distribution of labor incomes and, since they are negatively correlated with capital income, tends to reduce pre-tax income inequality. Third, tax changes have direct redistributive effects increasing or decreasing post-tax inequality for a given degree of pre-tax inequality.

We have illustrated the effects of fiscal policy by comparing the distributional consequences of financing an increase in public spending using alternative tax rates. This involves trade-offs between output and distribution, the nature of which depends upon the specific measure of distribution, the tax employed, and the use to which the tax revenues are applied. For example, an increase in a tax on capital that is spent on government consumption will be contractionary, and will reduce post-tax income inequality as well as welfare inequality. A tax on labor income spent on government consumption is also mildly contractionary, but is associated with higher post-tax income inequality and welfare inequality. An increase in either tax that is applied to transfers will give rise to qualitatively similar (but quantitatively dissimilar) tradeoffs. These results contrast with our analysis of an AK model [García-Peñalosa and Turnovsky, 2007]. There we found that when higher
taxes on capital and labor income are used to finance an investment subsidy, there is no conflict between productive efficiency and inequality, if the latter were measured by the distribution of welfare. In contrast, in the current setup and when tax revenues are used to finance utility-enhancing government consumption, fiscal changes that result in a greater (smaller) decline in output are associated with a reduction (increase) in welfare inequality.

Our analysis of income dynamics shows that there are two effects driving the evolution of income distribution over time: the immediate labor supply responses to tax changes and the ensuing transitional dynamics, which could have opposite signs. One surprising characteristic of the simulations is the mild nature of the distributional dynamics; most of the adjustment occurs on impact. This pattern is robust with respect to variations in parameters, and contrasts with the more pronounced transitional distributional dynamics obtained in our earlier study [Turnovsky and García-Peñalosa, 2008] in response to structural changes such as a productivity shock and a change in the rate of time preference. Taken together, these results suggest the following conclusion. With labor having complete flexibility to adjust, the more directly a structural change impinges on the consumer’s current budget constraint, [as does a lump-sum tax, a tax on labor income or consumption tax] the more fully labor adjusts on impact, the less are the subsequent transitional dynamics, and hence the less the impact on wealth inequality. In contrast, structural changes such as a tax on capital income or a general productivity increase, which impact directly on the firm and only indirectly on the consumer, are met by smaller immediate responses in labor supply, and a longer transitional adjustment, leading to a larger impact on wealth inequality.

We conclude with a number of caveats. First, the infinitely-lived agent framework abstracts from an important aspect of the evolution of the distribution of capital: the intergenerational transmission of wealth. As a result, our analysis does not examine the potential role of estate taxes in determining distribution. This question has been addressed in other work that complements our approach. Second, we have assumed no differences in wage rates across agents, implying that the distribution of earnings depends exclusively on the distribution of hours of work. Allowing for differences in the earning capacity of agents and examining how earning capacity and wealth interact to determine labor supply is clearly an important question and may well generate more transitional
dynamics in labor supply. Third, our assumptions, such as that of a balanced government budget, restrict the policies that we have been able to examine. Most notably, we have not examined the role of public debt, a question that we leave for future work. A model with debt and where the government is allowed to run a steady state deficit could be used to analyze, for example, the impact that a reduction in public expenditures aimed at reducing accumulated debt has on distribution. Finally, the fact that the distributions of wealth and income depend upon the initial jump in the labor supply implies that they are path-dependent. This means that the distributional consequences of a fiscal shock (or any structural shock) will depend upon whether it all occurs immediately, as we have been assuming, or is expected to take place over time.\footnote{This issue is addressed by Atolia, Chatterjee, and Turnovsky (2009) in the context of productivity changes. Using a Ramsey model, they show that whether a productivity increase occurs as a once-and–for-all event or gradually over a period of time can have an important effect on the distribution of wealth and therefore income. As a practical policy matter, this issue is probably more relevant in analyzing changes in levels (as in Atolia et al), which may well be announced to take place over time, rather than rates (considered here) where the full change is more likely to take place immediately.} This last issue may be relevant in a more detailed analysis of the distributional implications of observed tax changes in Europe and the US, since the current situation is the result of slowly diverging fiscal policies.
Table 1

Impact of an increase in government expenditure under alternative modes of financing

<table>
<thead>
<tr>
<th></th>
<th>Lump-sum tax $\tau$</th>
<th>Cons. tax $\tau_c$</th>
<th>Wage income tax $\tau_w$</th>
<th>Capital income tax $\tau_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\tilde{K}/\tilde{K}$</td>
<td>$\frac{\eta \tilde{L}}{1-\tilde{s}} &lt; 0$</td>
<td>$0$</td>
<td>$\frac{1+\eta}{1-\tilde{s}} \left[ \frac{\eta}{1+\eta} - \tilde{i} \right] &lt; 0$</td>
<td>$\frac{1}{1-\tilde{s}} \left( 1+\eta \right) \left[ \frac{\eta}{1+\eta} - \tilde{i} \right] + \frac{F_{\tilde{L}}}{F_{L\tilde{L}}} \left[ 1 + \eta (1 - g) \right] &lt; 0$</td>
</tr>
<tr>
<td>$d\tilde{L}/\tilde{L}$</td>
<td>$\frac{\eta \tilde{L}}{1-\tilde{s}} &lt; 0$</td>
<td>$0$</td>
<td>$\frac{1+\eta}{1-\tilde{s}} \left[ \frac{\eta}{1+\eta} - \tilde{i} \right] &lt; 0$</td>
<td>$\frac{1}{1-\tilde{s}} \left( 1+\eta \right) \left[ \frac{\eta}{1+\eta} - \tilde{i} \right] + \frac{\eta}{\tilde{F}_{KL}} \left[ (1-g)\tilde{F}_K - \delta \right]$</td>
</tr>
<tr>
<td>$d\tilde{Y}/\tilde{Y}$</td>
<td>$\frac{\eta \tilde{L}}{1-\tilde{s}} &lt; 0$</td>
<td>$0$</td>
<td>$\frac{1+\eta}{1-\tilde{s}} \left[ \frac{\eta}{1+\eta} - \tilde{i} \right] &lt; 0$</td>
<td>$\frac{1}{1-\tilde{s}} \left[ \frac{\eta}{1+\eta} - \tilde{i} \right] - \frac{1}{\tilde{F}_{KL}} \tilde{F}_K + \delta \eta &lt; 0$</td>
</tr>
<tr>
<td>$d\tilde{s}/\tilde{s}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\left( 1-\varepsilon \right)$</td>
</tr>
<tr>
<td>$d\tilde{\sigma}_k$</td>
<td>$-$</td>
<td>$0$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$d\tilde{\phi}$</td>
<td>$\frac{\eta}{1+\eta} &gt; 0$</td>
<td>$0$</td>
<td>$\frac{1}{\tilde{L}} \left[ \frac{\eta}{1+\eta} - \tilde{i} \right] &lt; 0$</td>
<td>$\frac{\eta}{1+\eta} + \frac{1}{(1+\eta)\tilde{L}} \left( 1-\varepsilon \right) + \frac{1}{\tilde{s}} \frac{\tilde{L} / \partial \tau_k}{\tilde{L}}$</td>
</tr>
<tr>
<td>$d\tilde{\psi}$</td>
<td>$\frac{\eta}{1+\eta} &gt; 0$</td>
<td>$0$</td>
<td>$\tilde{s}(1-\tilde{\phi}) + \frac{1}{\tilde{L}} \left[ \frac{\eta}{1+\eta} - \tilde{i} \right]$</td>
<td>$-(1-\tilde{\phi}) + \frac{\eta}{1+\eta} + \frac{1}{(1+\eta)\tilde{L}} \left( 1-\varepsilon \right) + \frac{1}{\tilde{s}} \frac{\tilde{L} / \partial \tau_k}{\tilde{L}}$</td>
</tr>
<tr>
<td>$d\tilde{\chi}$</td>
<td>$-\left( \frac{\eta^2 \tilde{L}^2}{(1+\eta)^2} \right) \frac{1}{(1-\tilde{s})} &lt; 0$</td>
<td>$0$</td>
<td>$\eta \frac{\tilde{L}}{\tilde{L}^2(1-\tilde{s})} \left[ \tilde{i} - \frac{\eta}{1+\eta} \right] &gt; 0$</td>
<td>$-\left( \frac{\eta}{1+\eta} \right) \frac{\tilde{L} / \partial \tau_k}{\tilde{L}}$</td>
</tr>
</tbody>
</table>
Table 2: Increase in government expenditure (US Economy)
(increase in \( g \) by 5 percentage points from 0.15 to 0.20)

<table>
<thead>
<tr>
<th></th>
<th>( \tilde{L} )</th>
<th>( \tilde{K} )</th>
<th>( \tilde{Y} )</th>
<th>( \sigma_k )</th>
<th>( \sigma_y )</th>
<th>( \sigma_{y,at} )</th>
<th>( \sigma_u )</th>
<th>( \sigma_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base:</strong></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.276 ), ( \tau_w = 0.224 ), ( \tau_c = 0.08 ), ( \tau = 0.148 )</td>
<td>0.307</td>
<td>2.580</td>
<td>0.990</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.276 ), ( \tau_w = 0.224 ), ( \tau_c = 0.08 ), ( \tau = 0.148 )</td>
<td>( \tau_k = 0.094 )</td>
<td>0.323</td>
<td>2.716</td>
<td>1.042</td>
</tr>
<tr>
<td>Lump-sum tax fin. ( \tau = 0.094 )</td>
<td>( \tau = 0.094 )</td>
<td>0.323</td>
<td>2.716</td>
<td>1.042</td>
<td>0.995</td>
<td>0.206</td>
<td>0.186</td>
<td>0.114</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.276 ), ( \tau_w = 0.224 ), ( \tau_c = 0.08 ), ( \tau = 0.148 )</td>
<td>( \tau_w = 0.308 )</td>
<td>0.298</td>
<td>2.509</td>
<td>0.962</td>
</tr>
<tr>
<td>Wage income tax fin. ( \tau_w = 0.308 )</td>
<td>( \tau = 0.094 )</td>
<td>0.298</td>
<td>2.509</td>
<td>0.962</td>
<td>1.0021</td>
<td>0.141</td>
<td>0.156</td>
<td>0.146</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.276 ), ( \tau_w = 0.224 ), ( \tau_c = 0.08 ), ( \tau = 0.148 )</td>
<td>( \tau_k = 0.419 )</td>
<td>0.312</td>
<td>2.509</td>
<td>0.962</td>
</tr>
<tr>
<td>Cap. income tax fin. ( \tau_k = 0.419 )</td>
<td>( \tau = 0.094 )</td>
<td>0.312</td>
<td>2.509</td>
<td>0.962</td>
<td>1.0395</td>
<td>0.188</td>
<td>0.102</td>
<td>0.132</td>
</tr>
</tbody>
</table>

Table 3: Increase in government expenditure (‘European’ Economy)
(increase in \( g \) by 5 percentage points from 0.20 to 0.25)

<table>
<thead>
<tr>
<th></th>
<th>( \tilde{L} )</th>
<th>( \tilde{K} )</th>
<th>( \tilde{Y} )</th>
<th>( \sigma_k )</th>
<th>( \sigma_y )</th>
<th>( \sigma_{y,at} )</th>
<th>( \sigma_u )</th>
<th>( \sigma_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base:</strong></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.36 ), ( \tau_w = 0.17 ), ( \tau_c = 0.21 ), ( \tau = 0.222 )</td>
<td>0.267</td>
<td>2.779</td>
<td>0.930</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.36 ), ( \tau_w = 0.17 ), ( \tau_c = 0.21 ), ( \tau = 0.222 )</td>
<td>( \tau = 0.161 )</td>
<td>0.284</td>
<td>2.953</td>
<td>0.988</td>
</tr>
<tr>
<td>Lump-sum tax fin. ( \tau = 0.161 )</td>
<td>( \tau = 0.161 )</td>
<td>0.284</td>
<td>2.953</td>
<td>0.988</td>
<td>0.996</td>
<td>0.0962</td>
<td>0.184</td>
<td>0.162</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.36 ), ( \tau_w = 0.17 ), ( \tau_c = 0.21 ), ( \tau = 0.222 )</td>
<td>( \tau_w = 0.455 )</td>
<td>0.252</td>
<td>2.627</td>
<td>0.879</td>
</tr>
<tr>
<td>Wage income tax fin. ( \tau_w = 0.455 )</td>
<td>( \tau = 0.161 )</td>
<td>0.252</td>
<td>2.627</td>
<td>0.879</td>
<td>1.0033</td>
<td>0.0157*</td>
<td>0.146</td>
<td>0.198</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \tau_k = 0.36 ), ( \tau_w = 0.17 ), ( \tau_c = 0.21 ), ( \tau = 0.222 )</td>
<td>( \tau_k = 0.319 )</td>
<td>0.272</td>
<td>2.082</td>
<td>0.850</td>
</tr>
<tr>
<td>Cap. income tax fin. ( \tau_k = 0.319 )</td>
<td>( \tau = 0.161 )</td>
<td>0.272</td>
<td>2.082</td>
<td>0.850</td>
<td>1.0270</td>
<td>0.0617</td>
<td>0.0835</td>
<td>0.180</td>
</tr>
</tbody>
</table>

*Corresponds to \( \varphi < 0 \)
Fig 1: Increase in G

Lump-sum tax financing

Wage income tax financing

Capital income tax financing

Pre-tax income distr

Wealth distribution

Welfare distr

Post-tax income distr
Fig 2: Increase in T

Consumption tax financing

Wage income tax financing

Capital income tax financing
Appendix

This Appendix is devoted to the derivation of the linear system. Linearizing eqs.(9a) and (9b) around the steady state yields the local dynamics for $K$ and $l$, 

\[
\begin{pmatrix}
\dot{K} \\
l
\end{pmatrix} = 
\begin{pmatrix}
a_{11} & a_{12} \\
\frac{a_{21}}{a_{22}} & a_{22}
\end{pmatrix}
\begin{pmatrix}
K - \bar{K} \\
l - \bar{l}
\end{pmatrix}
\]

where 

\[
a_{11} = (1 - g)F_K - \delta - F_{KL} \frac{l}{\eta} \frac{1 - \tau_w}{1 + \tau_c};
\]

\[
a_{12} = - \frac{F}{l} \left(1 - g\right) + \frac{1}{\eta} \frac{1 - \tau_w}{1 + \tau_c} + F_{Ll} \frac{l}{\eta} \frac{1 - \tau_w}{1 + \tau_c} < 0;
\]

\[
a_{21} = \frac{1}{H(l)} \left[F_{KK}(1 - \tau_g) - \left(1 - \gamma\right) \frac{F_{KL} - \theta \gamma \frac{F_K}{F}}{F_L} a_{11}\right];
\]

\[
a_{22} = \frac{1}{H(l)} \left[-F_{KL}(1 - \tau_g) - \left(1 - \gamma\right) \frac{F_{KL} - \theta \gamma \frac{F_K}{F}}{F_L} a_{12}\right]
\]

and 

\[
H \equiv \frac{1 - \gamma (1 + \eta)}{l} - \left(1 - \gamma\right) \frac{F_{KL}}{F_L} - \theta \gamma \frac{F_K}{F}.
\]

By direct calculation we can show that 

\[
\Gamma \equiv a_{11} a_{22} - a_{12} a_{21} = \frac{(1 - \tau_g)F_{KK}}{H} \left[\frac{(1 - g)F - \delta K}{L} + \frac{F}{\eta} (1 - \tau_c)\right]
\]

Substituting (13) and simplifying yields 

\[
\Gamma = \frac{(1 - \tau_g)(1 - \tau_c)F_{KK}F_L}{Hl\eta}
\]

In order for $\Gamma < 0$ and for the equilibrium to be a saddlepoint, we require $H > 0$, a condition that holds if $\theta$ is not too large, and we assume to be met. In fact, given the other parameter values that we assume, $H > 0$ for all $\theta < 3.12$, which clearly includes any reasonable specification of the parameters of the utility function.

We immediately see that $a_{12} < 0$. In order to determine the likely signs of the other elements, it is useful to express them in terms of dimensionless quantities such as the elasticity of substitution in production, $\varepsilon \equiv F_K F_L / FF_{KL}$ and $s \equiv F_K K / F$, the share of output going to capital.
Thus, using the steady-state equilibrium conditions, we may write

\[
a_{11} = \frac{(1-g)(\varepsilon-1)(\beta + \delta) - \delta(\varepsilon - \tilde{s})(1 - \tau_k)}{\varepsilon(1 - \tau_k)} \tag{A.1a}
\]

\[
a_{21} = \frac{F_{\gamma_k}}{HF_k} \left\{ (\beta + \delta) + a_{11} \frac{\tilde{s}}{1 - \tilde{s}} [1 - \gamma(1 + \theta \varepsilon)] \right\} \tag{A.1b}
\]

\[
a_{22} = - \frac{1}{H} \left\{ F_{\gamma_k} (1 - \tau_k) + a_{12} \left[ (1 - \gamma) \frac{F_{\gamma_k}}{F_L} - \theta \gamma \frac{F_{\gamma_k}}{F} \right] \right\} \tag{A.1c}
\]

the signs of which involve tradeoffs between \( \varepsilon \) and the other parameters. From (A.1c) we immediately find that our assumption \( \gamma < 0 \) is sufficient to ensure that \( a_{22} > 0 \), implying \( \mu - a_{22} < 0 \). In the case of the Cobb-Douglas production function, \( a_{11} < 0 \), while

\[
\text{sgn}(a_{21}) = \text{sgn}\left( \delta \tilde{s}[1 - \gamma(1 + \theta)] - (\beta + \delta) \right)
\]

This is certainly negative if either \( \delta = 0 \) (no depreciation) or \( \gamma = 0 \) (logarithmic utility). It is also negative for the parameter set employed in our numerical simulations: \( \beta = 0.04 \), \( \delta = 0.06 \), \( \gamma = -1.5 \), \( \theta = 0.30 \), and \( s = 0.36 \).
References


Benhabib, J. and A. Bisin, 2008. The distribution of wealth: Intergenerational transmission and redistributive policies.Unpublished working paper NYU.


Fig 1: Increase in G

Lump-sum tax financing

Wage income tax financing

Capital income tax financing
Fig 2: Increase in T

Consumption tax financing

Wage income tax financing

Capital income tax financing

Pre-tax income distr

Post-tax income distr

Wealth distr

Welfare distr